11.6: 6 converges by the Alternating Series Test since $f(x) = \frac{\ln x}{x} \Rightarrow f'(x) = \frac{1-\ln x}{x^2} < 0$ when $x > e \Rightarrow f(x)$ is decreasing $\Rightarrow u_n \geq u_{n+1}$; also $u_n \geq 0$ for $n \geq 1$ and $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{\ln n}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$

11.6: 10 diverges by the nth-Term Test since $\lim_{n\to\infty} \frac{\sqrt[3]{n+1}}{\sqrt{n}+1} = \lim_{n\to\infty} \frac{\sqrt[3]{1+\frac{1}{n}}}{1+\frac{1}{\sqrt{n}}} = 3 \neq 0$

11.6: 18 converges absolutely because the series $\sum_{n=1}^{\infty} \left|\frac{\sin n}{n^2}\right|$ converges by the Direct Comparison Test since $\left|\frac{\sin n}{n^2}\right| \leq \frac{1}{n^2}$

11.6: 39 converges conditionally since $\frac{\sqrt{n+1}-\sqrt{n}}{1} \cdot \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}}$ and $\left\{\frac{1}{\sqrt{n+1}+\sqrt{n}}\right\}$ is a decreasing sequence of positive terms which converges to $0 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}+\sqrt{n}}$ converges; but $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$ diverges by the Limit Comparison Test (part 1) with $\frac{1}{\sqrt{n}}$; a divergent p-series; $\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+1}+\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty} = \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{2}$

11.6: 40 diverges by the nth-Term Test since $\lim_{n\to\infty} (\sqrt{n^2+n} - n) = \lim_{n\to\infty} (\sqrt{n^2+n} - n) \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}+n} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{2} \neq 0$

11.6: 48 $|\text{error}| < |(-1)^4 t^4| = t^4 < 1$

11.7: 10 $\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty} \left|\frac{(x-1)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(x-1)^n}\right| < 1 \Rightarrow |x-1|\sqrt{\lim_{n\to\infty} \frac{n}{n+1}} < 1 \Rightarrow |x-1| < 1 \Rightarrow -1 < x-1 < 1 \Rightarrow 0 < x < 2$; when $x = 0$ we have $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/2}}$, a conditionally convergent series; when $x = 2$ we have $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$, a divergent series

a. the radius is 1; the interval of convergence is $0 \leq x < 2$

b. the interval of absolute convergence is $0 < x < 2$

c. the series converges conditionally at $x = 0$

11.7: 12 $\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty} \left|\frac{3^{n+1}x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n}\right| < 1 \Rightarrow 3|x| \lim_{n\to\infty} (\frac{1}{n+1}) < 1$ for all x

a. the radius is ∞; the series converges for all x

b. the series converges absolutely for all x

c. there are no values for which the series converges conditionally

11.7: 20 $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty}\left|\frac{\sqrt[n+1]{n+1}(2x+5)^{n+1}}{\sqrt[n]{n}(2x+5)^n}\right| < 1 \Rightarrow |2x+5|\lim_{n\to\infty}\left(\frac{\sqrt[n+1]{n+1}}{\sqrt[n]{n}}\right) < 1 \Rightarrow |2x+5|\left(\frac{\lim_{n\to\infty}\sqrt[t]{t}}{\lim_{n\to\infty}\sqrt[n]{n}}\right) < 1 \Rightarrow |2x+5| < 1 \Rightarrow -1 < 2x+5 < 1 \Rightarrow -3 < x < -2$; when $x=-3$ we have $\sum_{n=1}^{\infty}(-1)^n\sqrt{n}$, a divergence series since $\lim_{n\to\infty}\sqrt[n]{n}=1$; when $x=-2$ we have $\sum_{n=1}^{\infty}\sqrt[n]{n}$, a divergent series

a. the radius is $\frac{1}{2}$; the interval of convergence is $-3 \leq x < -2$

b. the interval of absolute convergence is $-3 < x < -2$

c. there are no values for which the series converges conditionally

11.7: 24 $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty}\left|\frac{(n+1)!(x-4)^{n+1}}{n!(x-4)^n}\right| < 1 \Rightarrow |x-4|\lim_{n\to\infty}(n+1) < 1 \Rightarrow$ only $x=4$ satisfies this inequality

a. the radius is 0; the series converges only for all $x=4$

b. the series converges absolutely only for $x=4$

c. there are no values for which the series converges conditionally

11.8: 12 $f(x)=(1-x)^{-1} \Rightarrow f'(x)=(1-x)^{-2},\ f''(x)=2(1-x)^{-3},\ f'''(x)=3!(1-x)^{-4} \Rightarrow \ldots f^{(k)}(x)=k!(1-x)^{-k-1};\ f(0)=1, f'(0)=1,\ f''(0)=2,\ f'''(0)=3!,\ldots,f^{(k)}(0)=k! \Rightarrow \frac{1}{1-x}=1+x+x^2+x^3+\ldots=\sum_{n=0}^{\infty}x^n$

11.8: 14 $\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!} \Rightarrow \sin\frac{x}{2}=\sum_{n=0}^{\infty}\frac{(-1)^n(\frac{x}{2})^{2n+1}}{(2n+1)!}=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2^{2n+1}(2n+1)!}=\frac{x}{2}-\frac{x^3}{2^3\cdot 3!}+\frac{x^5}{2^5\cdot 5!}+\ldots$

11.8: 21 $f(x)=x^3-2x+4 \Rightarrow f'(x)=3x^2-2,\ f''(x)=6x \Rightarrow f'''(x)=6 \Rightarrow f^{(n)}(x)=0$ if $n\geq 4$; $f(2)=8,\ f'(2)=10,\ f''(2)=12,\ f'''(2)=6,\ f^{(n)}(2)=0$ if $n\geq 4 \Rightarrow x^3-2x+4=8+10(x-2)+\frac{12}{2!}(x-2)^2+\frac{6}{3!}(x-2)^3=8+10(x-2)+6(x-2)^2+(x-2)^3$