

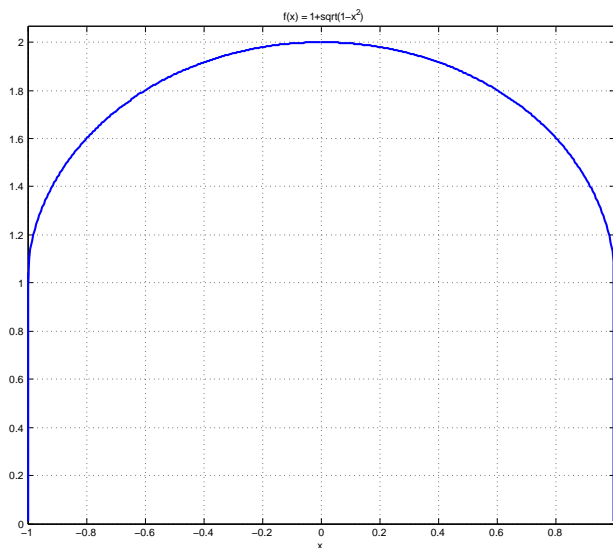
5.3: 2. $\int_{-1}^0 2x^3 dx$

5.3: 6. $\int_0^1 \sqrt{4-x^2} dx$

5.3: 14. (a) $\int_1^3 h(r) dr = \int_{-1}^3 h(r) dr - \int_{-1}^1 h(r) dr = 6 - 0 = 6$

(b) $-\int_3^1 h(u) du = -(-\int_1^3 h(u) du) = \int_1^3 h(u) du = 6$

5.3: 22. $y = 1 + \sqrt{1-x^2} \Rightarrow y-1 = \sqrt{1-x^2} \Rightarrow (y-1)^2 = 1-x^2 \Rightarrow x^2 + (y-1)^2 = 1$, a circle with center $(0, 1)$ and radius of 1 $\Rightarrow y = 1 + \sqrt{1-x^2}$ is the upper semicircle. The area of this semicircle is $A = 1/2 \cdot \pi r^2 = 1/2 \cdot \pi(1)^2 = \pi/2$. The area of the rectangular base is $A = lw = 2 \cdot 1 = 2$. Then the total area is $2 + \pi/2 \Rightarrow \int_{-1}^1 (1 + \sqrt{1-x^2}) dx = 2 + \pi/2$ square units.



5.3: 60.

$$av(f) = \frac{1}{1-(-2)} \int_{-2}^1 (t^2 - t) dt \quad (1)$$

$$= \frac{1}{3} \int_{-2}^1 t^2 dt - \frac{1}{3} \int_{-2}^1 t dt \quad (2)$$

$$= \frac{1}{3} \int_0^1 t^2 dt - \frac{1}{3} \int_{-2}^0 t^2 dt - \frac{1}{3} (1^2/2 - (-2)^2/2) \quad (3)$$

$$= 1/3(1^3/3) - 1/3((-2)^3/3) + 1/2 = 3/2 \quad (4)$$

5.4: 14. $\int_0^{\pi/3} 4 \sec u \tan u \, du = [4 \sec u]_0^{\pi/3} = 4 \sec(\pi/3) - 4 \sec 0 = 4(2) - 4(1) = 4$

5.4: 24. $\int_9^4 \frac{1-\sqrt{u}}{\sqrt{u}} \, du = \int_9^4 (u^{-1/2} - 1) \, du = [2\sqrt{u} - u]_9^4 = (2\sqrt{4} - 4) - (2\sqrt{9} - 9) = 3$

5.4: 28. (a) $\int_1^{\sin x} 3t^2 \, dt = [t^3]_1^{\sin x} = \sin^3 x - 1 \Rightarrow \frac{d}{dx} \left(\int_1^{\sin x} 3t^2 \, dt \right) = \frac{d}{dx} (\sin^3 x - 1) = 3 \sin^2 x \cos x$

(b) $\frac{d}{dx} \left(\int_1^{\sin x} 3t^2 \, dt \right) = (3 \sin^2 x) \frac{d}{dx} (\sin x) = 3 \sin^2 x \cos x$

5.4: 30. (a) $\int_0^{\tan \theta} \sec^2 y \, dy = [\tan y]_0^{\tan \theta} - 0 = \tan(\tan \theta) \Rightarrow \frac{d}{d\theta} \left(\int_0^{\tan \theta} \sec^2 y \, dy \right) = \frac{d}{d\theta} (\tan(\tan \theta)) = (\sec^2(\tan \theta)) \sec^2 \theta$

(b) $\frac{d}{d\theta} \left(\int_0^{\tan \theta} \sec^2 y \, dy \right) = (\sec^2(\tan \theta)) \frac{d}{d\theta} (\tan \theta) = (\sec^2(\tan \theta)) \sec^2 \theta$

5.4: 36. $y = \int_0^{\tan x} \frac{dt}{1+t^2} \Rightarrow \frac{dy}{dx} = \left(\frac{1}{1+\tan^2 x} \right) \left(\frac{d}{dx} (\tan x) \right) = (1/\sec^2 x)(\sec^2 x) = 1$

5.4: 43. The area of the rectangle bounded by the lines $y = 2$, $y = 0$, $x = \pi$, and $x = 0$ is 2π . The area under the curve $y = 1 + \cos x$ on $[0, \pi]$ is $\int_0^\pi (1 + \cos x) \, dx = [x + \sin x]_0^\pi = (\pi + \sin \pi) - (0 + \sin 0) = \pi$. Therefore the area of the shaded region is $2\pi - \pi = \pi$.