5.5: 4 Let $u = 1-\cos\frac{1}{2} \Rightarrow du = \frac{1}{2}\sin\frac{1}{2}dt \Rightarrow 2du = \sin\frac{1}{2}dt$. Then $\int(1-\cos\frac{t}{2}^2\sin\frac{t}{2})dt = \int 2u^2du = \frac{2}{3}u^3+C = \frac{2}{3}(1-\cos\frac{t}{2})^3+C$

5.5: 8 Let $u = y^4+4y^2+1 \Rightarrow du = (4y^3+8y)dy \Rightarrow 3du = 12(y^3+2y)dy$. Then $\int 12(y^4+4y^2+1)^2(y^3+2y)dy = \int 3u^2du = u^3+C = (y^4+4y^2+1)^3+C$

5.5: 12

a. Let $u = 5x+8 \Rightarrow du = 5dx \Rightarrow 1/5du = dx$. Then $\int \frac{dx}{\sqrt{5x+8}} = \int \frac{1}{5\sqrt{u}}du = 1/5\int u^{-1/2}du = 1/5(2u^{1/2})+C = 2/5u^{1/2}+C - 2/5\sqrt{5x+8}+C$

b. Let $u = \sqrt{5x+8} \Rightarrow du = 1/2(5x+8)^{-1/2}5dx \Rightarrow 2/5du = \frac{dx}{\sqrt{5x+8}}$. Then $\int \frac{dx}{\sqrt{5x+8}} = \int 2/5du = 2/5du + C = 2/5\sqrt{5x+8}+C$

5.5: 16 Let $u = 2-x \Rightarrow du = -dx \Rightarrow -du = dx$. Then $\int 3/(2-x)^2dx = \int 3(-du)/u^2 = -3\int u^{-2}du = -3(u^{-1}/(-1))+C = 3/(2-x)+C$

5.5: 22 Let $u = 1+\sqrt{x} \Rightarrow du = \frac{dx}{2\sqrt{x}} \Rightarrow 2du = dx/\sqrt{x}$. Then $\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}dx = \int u^3(2du) = 2(u^4/4)+C = 1/2(1+\sqrt{x})^4+C$

5.5: 36 Let $u = 2+\sin t \Rightarrow du = \cos t dt$. Then $\int \frac{6\cos t}{(2+\sin t)^3}dt = \int 6/u^3du = 6u^{(}-2)/(-2)+C = -3(2+\sin t)^{-2}+C$

5.5: 50

a. Let $u = x-1 \Rightarrow du = dx; v = \sin u \Rightarrow dv = \cos u du; w = 1+v^2 \Rightarrow dw = 2vdv \Rightarrow 1/2dw = vdv$. Then $\int\sqrt{1+\sin^2(x-1)}\sin(x-1)\cos(x-1)dx = \int\sqrt{1+\sin^2 u}\sin u\cos u du = \int v\sqrt{1+v^2}dv = \int\sqrt{w}/2dw = 1/3w^{3/2}+C = 1/3(1+v^2)^{3/2}+C = 1/3(1+\sin^2 u)^{3/2}+C = 1/3(1+\sin^2(x-1))^{3/2}+C$

b. Let $u = \sin(x-1) \Rightarrow du = \cos(x-1)dx; v = 1+u^2 \Rightarrow dv = 2udu \Rightarrow 1/2dv = udu$. Then $\int\sqrt{1+\sin^2(x-1)}\sin(x-1)\cos(x-1)dx = \int u\sqrt{1+u^2}du = \int\sqrt{v}/2dv = \frac{1}{2}\frac{2}{3}v^{3/2}+Cv^{3/2}/3+C = (1+u^2)^{3/2}/3+C = 1/3(1+\sin^2(x-1))^{3/2}+C$

c. Let $u = \sin^2(x-1) \Rightarrow du = 2\sin(x-1)\cos(x-1)dx; 1/2du = \sin(x-1)\cos(x-1)dx$. Then $\int\sqrt{1+\sin^2(x-1)}\sin(x-1)\cos(x-1)dx = \int\sqrt{u}/2du = \frac{1}{2}\frac{2u^{3/2}}{3}+C = 1/3(1+\sin^2(x-1))^{3/2}+C$

5.6: 8

a. Let $u = 1+v^{3/2} \Rightarrow du = 3v^{1/2}/2dv \Rightarrow \frac{20}{3}du = 10\sqrt{v}dv; v = 0 \Rightarrow u = 1, v = 1 \Rightarrow u = 2$. Then $\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}dv = \int_1^2 \frac{20}{3u^2}du = -\frac{20}{3}\left[\frac{1}{u}\right]_1^2 = -20/3[1/2-1/1] = 10/3$

b. Use the same substitution as in part (a); $v = 1 \Rightarrow u = 2, v = 4 \Rightarrow u = 1+4^{3/2} = 9$. Then $\int_1^4 \frac{10\sqrt{v}}{(1+v^{3/2})^2}dv = \int_2^9 \frac{20}{3u^2}du = -\frac{20}{3}\left[\frac{1}{u}\right]_2^9 = -20/3[1/9-1/2] = 70/27$

5.6: 12

a. Let $u = 2+\tan\frac{1}{2} \Rightarrow du = \frac{1}{2}\sec^2\frac{t}{2}dt \Rightarrow 2du = \sec^2\frac{t}{2}dt; t = \frac{-\pi}{2} \Rightarrow u = 2+\tan(\frac{-\pi}{2}) = 1, t = 0 \Rightarrow u = 2$. Then $\int_{-\pi/2}^0(2+\tan\frac{t}{2})\sec^2\frac{t}{2}dt = \int_1^2 u(2du) = [u^2]_1^2 = 2^2-1^2 = 3$

b. Use the same substitution as in part (a); $t = \frac{-pi}{2} \Rightarrow u = 1, t = \pi/2 \Rightarrow u = 3$. Then $\int_{-\pi/2}^{\pi/2}(2+\tan\frac{t}{2})\sec^2\frac{t}{2}dt = 2\int_1^3 udu = [u^2]_1^3 = 3^2-1^2 = 8$

5.6: 38 $AREA = A1 + A2$

A1: For the sketch given, $a = -2$ and $b = 0$; $f(x)-g(x) = (2x^3-x^2-5x)-(-x^2+3x) = 2x^3-8x \Rightarrow A1 = \int_{-2}^0(2x^3-8x)dx = \left[\frac{2x^4}{4}-\frac{8x^2}{2}\right]_{-2}^0 = 0-(8-16) = 8;$

A2: For the sketch given, $a = 0$ and $b = 2$; $f(x)-g(x) = (-x^2+3x)-(-2x^3-x^2-5x) = -2x^3+8x \Rightarrow A2 = \int_0^2(-2x^3+8x)dx = \left[-\frac{2x^4}{4}+\frac{8x^2}{2}\right]_0^2 = 0-(-8+16) = 8;$

Therefore, $AREA = A1 + A2 = 16$.

5.6: 40 $AREA = A1 + A2 + A3$

A1: For the sketch given, $a = -2$ and $b = 0$; $f(x) - g(x) = (x^3/3 - x) - x/3 = x^3/3 - 4x/3 \Rightarrow A1 = 1/3\int_{-2}^{0}(x^3 - 4x)dx = 1/3\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0} = 0 - 1/3(4-8) = 4/3$;

A2: For the sketch given, $a = 0$ and we find b by solving the equation $y = x^3/3 - x$ and $y = x/3$ simultaneously for x: $x^3/3 - x = x/3 \Rightarrow x^3/3 - 4x/3 = 0 \Rightarrow x(x-2)(x+2)/3 = 0 \Rightarrow x = -2, x = 0$, or $x = 2$ so $b = 2$; $f(x) - g(x) = (x/3) - (-x^3/3 - x) = -1/3(x^3 - 4x) \Rightarrow A2 = -1/3\int_0^2(x^3 - 4x)dx = 1/3\left[-2x^2 - \frac{x^4}{4}\right]_0^2 = 1/3(8-4) = 4/3$;

A3: For the sketch given, $a = 2$ and $b = 3$; $f(x) - g(x) = (x^3/3 - x) - x/3 = x^3/3 - 4x/3 \Rightarrow A3 = 1/3\int_2^3(x^3 - 4x)dx = 1/3\left[\frac{x^4}{4} - 2x^2\right]_2^3 = 1/3[(81/4 - 2\cdot 9) - (16/4 - 8)] = 1/3(81/4 - 14) = 25/12$;

Therefore, $AREA = A1 + A2 + A3 = 19/4$.

5.6: 46 Limits of integration: $7 - 2x^2 = x^2 + 4 \Rightarrow 3x^2 - 3 = 0 \Rightarrow 3(x-1)(x+1) = 0 \Rightarrow a = -1$ and $b = 1$; $f(x) - g(x) = (7 - 2x^2) - (x^2 + 4) = 3 - 3x^2 \Rightarrow A = \int_{-1}^{1}(3 - 3x^2)dx = 3\left[x - \frac{x^3}{3}]_{-1}^{1}\right] = 3[(1 - 1/3) - (-1 + 1/3)] = 6(2/3) = 4$

6.1: 2
a. $A = \pi(\text{radius})^2$ and radius $= \sqrt{x} \Rightarrow A(x) = \pi x$

b. $A = \text{width}\cdot\text{height}, \text{width} = \text{height} = 2\sqrt{x} \Rightarrow A(x) = 4x$

c. $A = (\text{side})^2$and diagonal $= \sqrt{2}(\text{side}) \Rightarrow A = \frac{(\text{diagonal})^2}{2}$; diagonal $= 2\sqrt{x} \Rightarrow A(x) = 2x$

d. $A = \frac{\sqrt{3}}{4}(\text{side})^2$and side $= 2\sqrt{x} \Rightarrow A(x) = \sqrt{3}x$

6.1: 10 $A(y) = 1/2(\text{leg})(\text{leg}) = 1/2[\sqrt{1-y^2} - (-\sqrt{1-y^2})]^2 = 1/2(2\sqrt{1-y^2})^2 = 2(1-y^2); c = -1, d = 1; V = \int_c^d A(y)dy = \int_{-1}^{1} 2(1-y^2)dy = 2[y - y^3/3]_{-1}^{1} = 4(1 - 1/3) = 8/3$

6.1: 44 $R(y) = 2 - y^{1/3}$ and $r(y) = 1 \Rightarrow V = \int_0^1 \pi([R(y)]^2 - [r(y)]^2)dy = \int_0^1 \pi([2 - y^{1/3}]^2 - 1)dy \int_0^1 \pi(4 - 4y^{1/3} + y^{2/3} - 1)dy = \pi[3y - 3y^{4/3} + 3y^{5/3}/5]_0^1 = \pi(3 - 3 + 3/5) = 3\pi/5$

6.1: 46
a. $r(y) = 0$ and $R(y) = 1 - y/2 \Rightarrow V = \int_0^2 \pi([R(y)]^2 - [r(y)]^2)dy = \pi\int_0^2(1 - y/2)^2dy = \pi\int_0^2(1 - y + y^2/4)dy = \pi[y - y^2/2 + y^3/12]_0^2 = \pi(2 - 4/2 + 8/12) = 2\pi/3$

b. $r(y) = 1$ and $R(y) = 2 - y/2 \Rightarrow V = \int_0^2 \pi([R(y)]^2 - [r(y)]^2)dy = \pi\int_0^2[(2 - y/2)^2 - 1]dy = \pi\int_0^2(4 - 2y + y^2/4 - 1)dy = \pi[3y - y^2 + y^3/12]_0^2 = \pi(6 - 4 + 8/12) = \pi(2 + 2/3) = 8\pi/3$

6.1: 50
a. A cross section has radius $r = \sqrt{2y}$ and area $\pi r^2 = 2\pi y$. The volume is $\int_0^5 2\pi y dy = \pi[y^2]_0^5 = 25\pi$.

b. $V(h) = \int A(h)dh$, so $\frac{dV}{dh} = A(h)$. Therefore $\frac{dV}{dt} = \frac{dV}{dh}\cdot\frac{dh}{dt} = A(h)\cdot\frac{dh}{dt}$, so $\frac{dh}{dt} = \frac{1}{A(h)}\cdot\frac{dV}{dt}$. For $h = 4$, the area is $2\pi(4) = 8\pi$, so $\frac{dh}{dt} = \frac{1}{8\pi}\cdot 3\frac{\text{unit}^3}{\text{sec}}$.