6.2: 4 For the sketch given, $c = 0$, $d = \sqrt{3}$; $V = \int_c^d 2\pi(\text{shell radius})(\text{shell height})dy = \int_0^{\sqrt{3}} 2\pi y \cdot (y^2)dy = 2\pi \int_0^{\sqrt{3}} y^3 dy = 2\pi \left[\frac{y^4}{4}\right]_0^{\sqrt{3}} = 9\pi/2$.

6.2: 10 $a = 0$, $b = 1$; $V = \int_a^b 2\pi(\text{shell radius})(\text{shell height})dx = \int_0^1 2\pi x[(2-x^2)-x^2]dx = 2\pi \int_0^1 x(2-2x^2)dx = 4\pi \int_0^1 (x-x^3)dx = 4\pi \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 4\pi(1/2 - 1/4) = \pi$.

6.2: 22 $c = 0$, $d = 1$; $V = \int_c^d 2\pi(\text{shell radius})(\text{shell height})dy = \int_0^1 2\pi y[(2-y)-y^2]dy = 2\pi \int_0^1 (2y-y^2-y^3)dy = 2\pi \left[y^2 - \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = 2\pi(1-1/3-1/4) = \frac{\pi}{6}(12-4-3) = 5\pi/6$.

6.2: 24

a. $V = \int_c^d 2\pi(\text{shell radius})(\text{shell height})dy = \int_0^2 2\pi y \left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \int_0^2 2\pi y \left(y^2 - \frac{y^4}{4}\right) dy = \int_0^2 2\pi \left(y^3 - \frac{y^5}{4}\right) dy = 2\pi \left[\frac{y^4}{4} - \frac{y^6}{24}\right]_0^2 = 2\pi \left(\frac{2^4}{4} - \frac{2^6}{24}\right) = 32\pi(1/4 - 4/24) = 8\pi/3$.

b. $V = \int_c^d 2\pi(\text{shell radius})(\text{shell height})dy = \int_0^2 2\pi(2-y) \left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \int_0^2 2\pi(2-y) \left(y^2 - \frac{y^4}{4}\right) dy = \int_0^2 2\pi \left(2y^2 - \frac{y^4}{2} - y^3 + \frac{y^5}{4}\right) dy = 2\pi \left[\frac{2y^3}{3} - \frac{y^5}{10} - \frac{y^4}{4} + \frac{y^6}{24}\right]_0^2 = 2\pi\left(16/3 - 32/10 - 16/4 + 64/24\right) = 8\pi/5$.

c. $V = \int_c^d 2\pi(\text{shell radius})(\text{shell height})dy = \int_0^2 2\pi(5-y) \left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \int_0^2 2\pi(5-y) \left(y^2 - \frac{y^4}{4}\right) dy = \int_0^2 2\pi \left(5y^2 - \frac{5y^4}{4} - y^3 + \frac{y^5}{4}\right) dy = 2\pi \left[\frac{5y^3}{3} - \frac{5y^5}{20} - \frac{y^4}{4} + \frac{y^6}{24}\right]_0^2 = 2\pi\left(40/3 - 160/20 - 16/4 + 64/24\right) = 8\pi$.

d. $V = \int_c^d 2\pi(\text{shell radius})(\text{shell height})dy = \int_0^2 2\pi(y+5/8) \left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \int_0^2 2\pi(y+5/8) \left(y^2 - \frac{y^4}{4}\right) dy = \int_0^2 2\pi \left(y^3 - \frac{y^5}{4} - \frac{5y^2}{8} - \frac{5y^4}{32}\right) dy = 2\pi \left[\frac{y^4}{4} - \frac{y^5}{24} - \frac{5y^3}{24} + \frac{5y^5}{160}\right]_0^2 = 2\pi\left(16/4 - 64/24 + 40/24 - 160/160\right) = 4\pi$.

6.2: 34

a. $V = \int_c^d \pi[R^2(y) - r^2(y)]dy = \int_1^2 \pi(1/y^4 - 1/16)dy = \pi\left[-\frac{y^{-3}}{3} - \frac{y}{16}\right]_1^2 = \pi[(-1/24 - 1/8) - (-1/3 - 1/16)] = \frac{11\pi}{48}$.

b. $V = \int_a^b 2\pi(\text{shell radius})(\text{shell height})dx = \int_{1/4}^1 2\pi x(\frac{1}{\sqrt{x}} - 1)dx = 2\pi \int_{1/4}^1 (x^{1/2} - x)dx = 2\pi \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_{1/4}^1 = \frac{11\pi}{48}$.

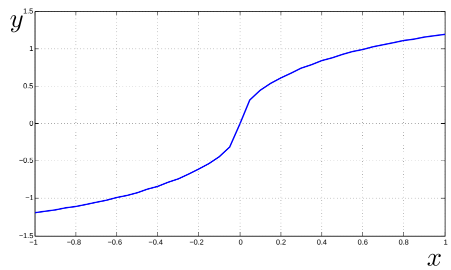
6.3: 2 $\frac{dx}{dt} = -\sin t$ and $\frac{dy}{dt} = 1 + \cos t \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(-\sin t)^2 + (1+\cos t)^2} = \sqrt{2 + 2\cos t} \Rightarrow$ Length $= \int_0^\pi \sqrt{2+2\cos t}\,dt = \sqrt{2}\int_0^\pi \sqrt{\left(\frac{1-\cos t}{1-\cos t}\right)(2+2\cos t)}dt = \sqrt{2}\int_0^\pi \sqrt{\frac{\sin^2 t}{1-\cos t}}dt = \sqrt{2}\int_0^\pi \frac{\sin t}{\sqrt{1-\cos t}}dt$ (since $\sin t \geq 0$ on $[0,\pi]$); $[u = 1 - \cos t \Rightarrow du = \sin t\,dt;\ t = 0 \Rightarrow u = 0,\ t = \pi \Rightarrow u = 2] \to \sqrt{2}\int_0^2 u^{-1/2}du = \sqrt{2}[2u^{1/2}]_0^2 = 4$.

6.3: 12 $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{4}(y^4 - 2 + y^{-4}) \Rightarrow L = \int_2^3 \sqrt{1 + \frac{1}{4}(y^4 - 2 + y^{-4})}dy = \int_2^3 \sqrt{\frac{1}{4}(y^4 + 2 + y^{-4})}dy = \frac{1}{2}\int_2^3 \sqrt{(y^2 + y^{-2})^2}dy = \frac{1}{2}\int_2^3 (y^2 + y^{-2})^2 dy = \frac{1}{2}\left[\frac{y^3}{3} - y^{-1}\right]_2^3 = \frac{13}{4}$.

6.3: 24

a. $\frac{dx}{dy} = \sqrt{\sec^2 y - 1} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \sec^2 y - 1 \Rightarrow L = \int_{-\pi/3}^{\pi/4} \sqrt{1 + (\sec^2 y - 1)}dy = \int_{-\pi/3}^{\pi/4} |\sec y| dy = \int_{-\pi/3}^{\pi/4} \sec y\,dy$.

$x = \int_0^y \sqrt{\sec^2 y - 1}dt$



b.

c. $L \approx 2.20$

6.3: 28 a. $\left(\frac{dx}{dy}\right)^2$ corresponds to $\frac{1}{y^4}$ here, so take $\frac{dx}{dy}$ as $\frac{1}{y^2}$. Then $x = -1/y + C$ and, since (0, 1) lies on the curve, $C = 1$. So $y = \frac{1}{1-x}$.

b. Only one. We know the derivative of the function and the value of the function at one value of x.

6.3: 30 $x = a(\theta - \sin\theta) \Rightarrow \frac{dx}{dy} = a(1-\cos\theta) \Rightarrow \left(\frac{dx}{d\theta}\right)^2 = a^2(1 - 2\cos\theta + \cos^2\theta)$ and $y = a(1-\cos\theta) \Rightarrow \frac{dy}{d\theta} = a\sin\theta \Rightarrow \left(\frac{dy}{d\theta}\right)^2 = a^2\sin^2\theta \Rightarrow L = \int_0^{2\pi}\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}d\theta = \int_0^{2\pi}\sqrt{2a^2(1-\cos\theta)}d\theta = a\sqrt{2}\int_0^{2\pi}\sqrt{2}\sqrt{\frac{1-\cos\theta}{2}}d\theta = 2a\int_0^{2\pi}|\sin\frac{\theta}{2}|d\theta = 2a\int_0^{2\pi}\sin\frac{\theta}{2}d\theta = -4a[\cos\frac{\theta}{2}]_0^{2\pi} = 8a$.

6.4: 4 Let the rods have lengths $x = L$ and $y = 2L$. The center of mass of each rod is in its center (see Example 1). The rod system is equivalent to two point masses located at the centers of the rods at coordinates $(L/2, 0)$ and $(0, L)$. Therefore $\overline{x} = \frac{\frac{L}{2}\cdot m + 0}{m + 2m} = L/6$ and $\overline{y} = \frac{0\cdot m + L\cdot 2m}{m+2m} = \frac{2L}{3} \Rightarrow (L/6, 2L/3)$ is the center of mass location.

6.4: 8 $M_0 = \int_0^4 x\left(2 - \frac{x}{2}\right)dx = \int_0^4\left(2x - \frac{x^2}{2}\right)dx = \left[x^2 - \frac{x^3}{12}\right]_0^4 = (16 - 64/12) = 32/3$; $M = \int_0^4\left(2 - \frac{x}{4}\right)dx = \left[2x - \frac{x^2}{8}\right]_0^4 = 8 - 16/8 = 6 \Rightarrow \overline{x} = \frac{M_0}{M} = \frac{16}{9}$.

6.4: 12 $M_0 = \int_0^1 x(x+1)dx + \int_1^2 2xdx = \int_0^1(x^2+x)dx + \int_1^2 2xdx = \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1 + [x^2]_1^2 = (1/3 + 1/2) + (4-1) = 23/6$; $M = \int_0^1(x+1)dx + \int_1^2 2dx = \left[\frac{x^2}{2} + x\right]_0^1 + [2x]_1^2 = (1/2+1) + (4-2) = 7/2 \Rightarrow \overline{x} = \frac{M_0}{M} = \frac{23}{21}$.

6.4: 36 $y = x^3 \Rightarrow dy = 3x^2dx \Rightarrow dx = \sqrt{(dx)^2 + (3x^2dx)^2} = \sqrt{1+9x^4}dx$; $M_x = \delta\int_0^1 x^3\sqrt{1+9x^4}dx$; $[u = 1 + 9x^4 \Rightarrow du = 36x^3dx \Rightarrow \frac{1}{36}du = x^3dx;\ x = 0 \Rightarrow u = 1,\ x = 1 \Rightarrow u = 10] \rightarrow M_s = \delta\int_1^{10}\frac{1}{36}u^{1/2}du = \frac{\delta}{36}\left[\frac{2}{3}u^{3/2}\right]_1^{10} = \frac{\delta}{54}(10^{3/2} - 1)$.