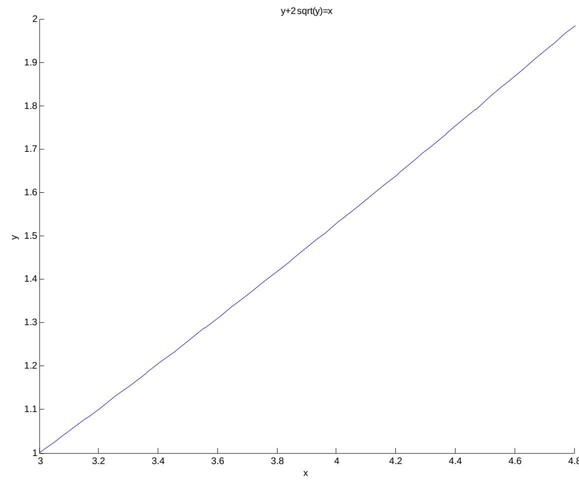
6.5: 6 a. $\frac{dx}{dy} = 1 + y^{-1/2} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \left(1 + y^{-1/2}\right)^2 \Rightarrow S = 2\pi \int_1^2 (y + 2\sqrt{y})\sqrt{1 + (1 + y^{-1/2})^2}dx.$



b.

c. $S \approx 51.33.$

6.5: 12 $y = \frac{x}{2} + \frac{1}{2} \Rightarrow x = 2y - 1 \Rightarrow \frac{dx}{dy} = 2; S = \int_c^d 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}dy = \int_1^2 2\pi(2y-1)\sqrt{1+4}dy = 2\pi\sqrt{5}\int_1^2 (2y-1)dy = 2\pi\sqrt{5}$; Geometry formula: $r_1 = 1,\ r_2 = 3$, slant height $= \sqrt{(2-1)^2 + (3-1)^2} = \sqrt{5} \Rightarrow$ Frustum surface area $= \pi(1+3)\sqrt{5} = 4\pi\sqrt{5}$ in agreement with the integral value.

6.5: 14 $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4x} \Rightarrow S = \int_{3/4}^{15/4} 2\pi\sqrt{x}\sqrt{1 + \frac{1}{4x}}dx = 2\pi\int_{3/4}^{15/4}\sqrt{x + \frac{1}{4}}dx = 2\pi\left[\frac{2}{3}\left(x + \frac{1}{4}\right)^{3/2}\right]_{3/4}^{15/4} = \frac{28\pi}{3}$

6.5: 36 $\frac{dx}{dt} = a(1 - \cos t)$ and $\frac{dy}{dt} = a\sin t \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{[a(1-\cos t)]^2 + (a\sin t)^2} = a\sqrt{2}\sqrt{1-\cos t} \Rightarrow S = \int 2\pi y ds = \int_0^{2\pi} 2\pi a(1-\cos t)\cdot a\sqrt{2}\sqrt{1-\cos t} = 2\sqrt{2}\pi a^2\int_0^{2\pi}(1-\cos t)^{3/2}dt.$

6.5: 38 $\frac{dx}{dt} = h$ and $\frac{dy}{dt} = r \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{h^2 + r^2} \Rightarrow S = \int 2\pi ds = \int_0^1 2\pi rt\sqrt{h^2+r^2}dt = 2\pi r\sqrt{h^2+r^2}\int_0^1 tdt = 2\pi rt\sqrt{h^2+r^2}\left[\frac{t^2}{2}\right]_0^1 = \pi r\sqrt{h^2+r^2}$. Check: slant height is $\sqrt{h^2+r^2} \Rightarrow$ Area is $\pi r\sqrt{h^2+r^2}$.

6.6: 7 The force required to haul up the rope is equal to the rope's weight, which varies steadily and is proportional to x, the length of the rope still hanging: $F(x) = 0.624x$. The work done is: $W = \int_0^{50} F(x)dx = \int_0^{50} 0.624xdx = 780J.$

6.6: 9 The force required to lift the cable is equal to the weight of the cable paid out: $F(x) = (4.5)(180 - x)$ where x is the position of the car off the first floor. The work done is: $W = \int_0^{180} F(x) = 4.5 \int_0^{180} (180 - x)dx = 72,900\text{ft} \cdot \text{lb}$.