6.7: 17 When the water reaches the top of the tank, the force on the movable side is $\int_{-2}^{0}(62.4)(2\sqrt{4-y^2})(-y)dy = (62.4)\int_{-2}^{0}(4-y^2)^{1/2}(-2y)dy = (62.4)\left[\frac{2}{3}(4-y^2)^{3/2}\right]_{-2}^{0} = 332.8\text{ft}\cdot\text{lb}$. The force compressing the spring is $F = 100x$, so when the tank is full we have $332.8 = 100x \Rightarrow x \approx 3.33\text{ft}$. Therefore the movable end does not reach the required 5ft to allow drainage $\Rightarrow$ the tank will overflow.

7.1: 14 Step 1: $y = x^2 \Rightarrow x = -\sqrt{y}$, since $x \le 0$ Step 2: $y = -\sqrt{x} = f^{-1}(x)$

7.1: 16 Step 1: $y = x^2 - 2x + 1 \Rightarrow y = (x-1)^2 \Rightarrow \sqrt{y} = x - 1$, since $x \ge 1 \Rightarrow x = 1 + \sqrt{y}$ Step 2: $y = 1 + \sqrt{x} = f^{-1}(x)$

7.1: 29 a. $f(g(x)) = (\sqrt[3]{x})^3 = x,\ g(f(x)) = \sqrt[3]{x^3} = x$

b.

$$\begin{aligned} f'(x) &= 3x^2 \Rightarrow f'(1) = 3,\ f'(-1) = 3; & (1)\\ g'(x) &= \frac{1}{3}x^{-2/3} \Rightarrow g'(1) = \frac{1}{3},\ g'(-1) = \frac{1}{3} & (2) \end{aligned}$$

c. The line $y = 0$ is tangent to $f(x) = x^3$ at $(0,0)$; the line $x = 0$ is tangent to $g(x) = \sqrt[3]{x}$ at $(0,0)$

7.1: 32 $\frac{df}{dx} = 2x - 4 \Rightarrow \left.\frac{df^{-1}}{dx}\right|_{x=f(5)} = \left.\frac{1}{\frac{df}{dx}}\right|_{x=5} = \frac{1}{6}$

7.1: 34 $\left.\frac{dg^{-1}}{dx}\right|_{x=0} = \left.\frac{dg^{-1}}{dx}\right|_{x=f(0)} = \left.\frac{1}{\frac{dg}{dx}}\right|_{x=0} = \frac{1}{2}$

7.2: 2 a. $\ln\frac{1}{125} = \ln 1 - 3\ln 5 = -3\ln 5$

b. $\ln 9.8 = \ln\frac{40}{5} = \ln 7^2 - \ln 5 = 2\ln 7 - \ln 5$

c. $\ln 7\sqrt{7} = \ln 7^{3/2} = \frac{3}{2}\ln 7$

d. $\ln 1225 = \ln 35^2 = 2\ln 35 = 2\ln 5 + 2\ln 7$

e. $\ln 0.056 = \ln\frac{7}{125} = \ln 7 - \ln 5^3 = \ln 7 - 3\ln 5$

f. $\frac{\ln 35 + \ln\frac{1}{2}}{\ln 25} = \frac{\ln 5 + \ln 7 - \ln 7}{2\ln 5} = \frac{1}{2}$

7.2: 22 $y = \frac{x\ln x}{1+\ln x} \Rightarrow y' = \frac{(1+\ln x)(\ln x + x\cdot\frac{1}{x}) - (x\ln x)(\frac{1}{x})}{(1+\ln x)^2} = \frac{(1+\ln x)^2 - \ln x}{(1+\ln x)^2} = 1 - \frac{\ln x}{(1+\ln x)^2}$

7.2: 38 $\int_{-1}^{0}\frac{3}{3x-2}dx = [\ln|3x-2|]_{-1}^{0} = \ln 2 - \ln 5 = \ln\frac{2}{5}$

7.2: 52 Let $u = \cos 3x \Rightarrow du = -3\sin 3x dx \Rightarrow -2du = 6\sin 3x dx$; $x = 0 \Rightarrow u = 1$ and $x = \frac{\pi}{12} \Rightarrow u = \frac{1}{\sqrt{2}}$; $\int_0^{\pi/12} 6\tan 3x dx = \int_0^{\pi/12}\frac{6\sin 3x}{\cos 3x}dx = -2\int_1^{1/\sqrt{2}}\frac{du}{u} = -2[\ln|u|]_1^{1/\sqrt{2}} = -2\ln\frac{1}{\sqrt{2}} - \ln 1 = 2\ln\sqrt{2} = \ln 2$

7.2: 58 $y = \sqrt{\frac{1}{t(t+1)}} = [t(t+1)]^{-1/2} \Rightarrow \ln y = \frac{1}{2}[\ln t + \ln(t+1)] \Rightarrow \frac{1}{y}\frac{dy}{dt} = -\frac{1}{2}\left(\frac{1}{t} + \frac{1}{t+1}\right) \Rightarrow \frac{dy}{dt} = -\frac{1}{2}\sqrt{\frac{1}{t(t+1)}}\left[\frac{2t+1}{t(t+1)}\right] = -\frac{2t+1}{2(t^2+t)^{3/2}}$

7.3: 4 a. $\ln\left(e^{\sec\theta}\right) = (\sec\theta)(\ln e) = \sec\theta$

b. $\ln e^{(e^x)} = (e^x)(\ln e) = e^x$

c. $\ln\left(e^{2\ln x}\right) = \ln\left(e^{\ln x^2}\right) = \ln x^2 = 2\ln x$

7.3: 26 $y = \ln\left(3\theta e^{-\theta}\right) = \ln 3 + \ln\theta + \ln e^{-\theta} = \ln 3 + \ln\theta - \theta \Rightarrow \frac{dy}{d\theta} = \frac{1}{\theta} - 1$

7.3: 50 Let $u = -r^{1/2} \Rightarrow du = -\frac{1}{2}r^{-1/2}dr \Rightarrow -2du = r^{-1/2}dr$; $\int\frac{e^{-\sqrt{r}}}{\sqrt{r}}dr = -2\int e^u du = -2e^{-r^{1/2}} + C = -2e^{-\sqrt{r}} + C$