7.4: 6 a. $\frac{\log_9 x}{\log_3 x} = \frac{\ln x}{\ln 9} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{2\ln 3} \cdot \frac{\ln 3}{\ln x} = \frac{1}{2}$

b. $\frac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x} = \frac{\ln x}{\ln \sqrt{10}} \div \frac{\ln x}{\ln \sqrt{2}} = \frac{\ln x}{\frac{1}{2}\ln 10} \cdot \frac{\frac{1}{2}\ln 2}{\ln x} = \frac{\ln 2}{\ln 10}$

c. $\frac{\log_a b}{\log_b a} = \frac{\ln b}{\ln a} \div \frac{\ln a}{\ln b} = \frac{\ln b}{\ln a} \cdot \frac{\ln b}{\ln a} = \left(\frac{\ln b}{\ln a}\right)^2$

7.4: 8 $8^{\log_8(3)} - e^{\ln 5} = x^2 - 7^{\log_7(3x)} \Rightarrow 3 - 5 = x^2 - 3x \Rightarrow 0 = x^2 - 3x + 2 = (x-1)(x-2) \Rightarrow x = 1$ or $x = 2$

7.4: 20 $y = 3^{\tan\theta}\ln 3 \Rightarrow \frac{dy}{d\theta} = (3^{\tan\theta}\ln 3)(\ln 3)\sec^2\theta = 3^{\tan\theta}(\ln 3)^2\sec^2\theta$

7.4: 64 $\int_1^e \frac{2\ln 10(\log_1 0x)}{x}dx = \int_1^e \frac{(\ln 10)(2\ln x)}{(\ln 10)}\frac{1}{x}dx = [(\ln x)^2]_1^e = (\ln e)^2 - (\ln 1)^2 = 1$

7.4: 72 $\int_1^{e^x} \frac{1}{t}dt = [\ln|t|]_1^{e^x} = \ln e^x - \ln 1 = x\ln e = x$

7.5: 6 $V(t) = V_0 e^{-t/40} \Rightarrow 0.1V_0 = V_0 e^{-t/40}$ when the voltage is 10% of its original value $\Rightarrow t = -40\ln(0.1) \approx 92.1$ sec

7.5:8 $y = y_0 e^{kt}$ and $y(3) = 10,000 \Rightarrow 10,000 = y_0 e^{3k}$; also $y(5) = 40,000 = y_0 e^{5k}$. Therefore $y_0 e^{5k} = 4y_0 e^{3k} \Rightarrow e^{5k} = 4e^{3k} \Rightarrow e^{2k} = 4 \Rightarrow k = \ln 2$. Thus, $y = y_0 e^{(\ln 2)t} \Rightarrow 10,000 = y_0 e^{3\ln 2} = y_0 e^{\ln 8} \Rightarrow 10,000 = 8y_0 \Rightarrow y_0 = \frac{10,000}{8} = 1250$

7.5: 18 $A = A_0 e^{kt}$ and $\frac{1}{2}A_0 = A_0 e^{139k} \Rightarrow \frac{1}{2} = e^{139k} \Rightarrow k = \frac{\ln(0.5)}{139} \approx -0.00499$; then $0.05A_0 = A_0 e^{-0.00499t} \Rightarrow t = \frac{\ln 0.05}{-0.00499} \approx 600$ days

7.5: 25 From Example 5, the half-life of carbon-14 is 5700 yr $\Rightarrow \frac{1}{2}c_0 = c_0 e^{-k(5700)} \Rightarrow k = \frac{\ln 2}{5700} \approx 0.0001216 \Rightarrow c = c_0 e^{-0.0001216t} \Rightarrow (0.445)c_0 = c_0 e^{-0.0001216t} \Rightarrow t = \frac{\ln(0.445)}{-0.0001216} \approx 6659$ years

7.6: 4 a. same, $\lim_{x\to\infty} \frac{x^2+\sqrt{x}}{x^2} = \lim_{x\to\infty}\left(1 + \frac{1}{x^{3/2}}\right) = 1$

b. same, $\lim_{x\to\infty} \frac{10x^2}{x^2} = \lim_{x\to\infty} 10 = 10$

c. slower, $\lim_{x\to\infty} \frac{x^2 e^{-x}}{x^2} = \lim_{x\to\infty} \frac{1}{e^x} = 0$

d. slower, $\lim_{x\to\infty} \frac{\log_{10} x^2}{x^2} = \lim_{x\to\infty} \frac{\left(\frac{\ln x^2}{\ln 10}\right)}{x^2} = \frac{1}{\ln 10}\lim_{x\to\infty} \frac{2\ln x}{x^2} = \frac{2}{\ln 10}\lim_{x\to\infty} \frac{1/x}{2x} = \frac{1}{\ln 10}\lim_{x\to\infty} \frac{1}{x^2} = 0$

e. faster, $\lim_{x\to\infty} \frac{x^3-x^2}{x^2} = \lim_{x\to\infty}(x-1) = \infty$

f. slower, $\lim_{x\to\infty} \frac{(1/10)^x}{x^2} = \lim_{x\to\infty} \frac{1}{10^x x^2} = 0$

g. faster, $\lim_{x\to\infty} \frac{(1.1)^x}{x^2} = \lim_{x\to\infty} \frac{(\ln 1.1)(1.1)^x}{2x} = \lim_{x\to\infty} \frac{(\ln 1.1)^2(1.1)^x}{2} = \infty$

h. same, $\lim_{x\to\infty} \frac{x^2+100x}{x^2} = \lim_{x\to\infty}\left(1 + \frac{100}{x}\right) = \infty$

7.6: 6 a. same, $\lim_{x\to\infty} \frac{\log_2 x^2}{\ln x} = \lim_{x\to\infty} \frac{\frac{\ln x^2}{\ln 2}}{\ln x} = \frac{1}{\ln 2}\lim_{x\to\infty} \frac{\ln x^2}{\ln x} = \frac{1}{\ln 2}\lim_{x\to\infty} \frac{2\ln x}{\ln x} = \frac{1}{\ln 2}\lim_{x\to\infty} 2 = \frac{2}{\ln 2}$

b. same, $\lim_{x\to\infty} \frac{\log_{10} 10x}{\ln x} = \lim_{x\to\infty} \frac{\left(\frac{\ln 10x}{\ln x}\right)}{\ln x} = \frac{1}{\ln 10}\lim_{x\to\infty} \frac{\ln 10x}{\ln x} = \frac{1}{\ln 10}\lim_{x\to\infty} \frac{\frac{10}{10x}}{\frac{1}{x}} = \frac{1}{\ln 10}\lim_{x\to\infty} 1 = \frac{1}{\ln 10}$

c. slower, $\lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{\ln x} = \lim_{x\to\infty} \frac{1}{\sqrt{x}\ln x} = 0$

d. slower, $\lim_{x\to\infty} \frac{\frac{1}{x^2}}{\ln x} = \lim_{x\to\infty} \frac{1}{x^2\ln x} = 0$

e. faster, $\lim_{x\to\infty} \frac{x-2\ln x}{\ln x} = \lim_{x\to\infty} \left(\frac{x}{\ln x} - 2\right) = \left(\lim_{x\to\infty} \frac{x}{\ln x}\right) - 2 = \left(\lim_{x\to\infty} \frac{1}{\frac{1}{x}}\right) - 2 = (\lim_{x\to\infty} x) - 2 = \infty$

f. slower, $\lim_{x\to\infty} \frac{e^{-x}}{\ln x} = \lim_{x\to\infty} \frac{1}{e^x \ln x} = 0$

g. slower, $\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x} = \lim_{x\to\infty} \frac{\left(\frac{1/x}{\ln x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1}{\ln x} = 0$

h. same, $\lim_{x\to\infty} \frac{\ln(2x+5)}{\ln x} = \lim_{x\to\infty} \frac{\frac{2}{2x+5}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{2x}{2x+5} = \lim_{x\to\infty} \frac{2}{2} = \lim_{x\to\infty} 1 = 1$