Math 1920 Homework 1 Selected Solutions

13.1

PQ2)

$\|-3\mathbf{a}\| = |-3|\,\|\mathbf{a}\| = 3 \cdot 5 = 15.$

44)

If $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$ then

$$\mathbf{e_v} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{4\mathbf{i} + 3\mathbf{j}}{\sqrt{4^2 + 3^2}} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$$

consequently the vector we want is

$$3\mathbf{e_v} = \frac{12}{5}\mathbf{i} + \frac{9}{5}\mathbf{j}$$

64)

We resolve the vectors into horizontal and vertical components

force	vertical component	horizontal component
$\mathbf{F}_1$	$\|\mathbf{F}_1\|$	0
$\mathbf{F}_2$	$\sin(45^\circ)\,\|\mathbf{F}_2\|$	$\cos(45^\circ)\,\|\mathbf{F}_2\|$
20	$\sin(30^\circ) \cdot 20$	$\cos(30^\circ) \cdot 20$

As the forces are balanced we know

$$\|\mathbf{F}_1\| = \sin(45^\circ)\,\|\mathbf{F}_2\| + \sin(30^\circ) \cdot 20, \quad \cos(45^\circ)\,\|\mathbf{F}_2\| = \cos(30^\circ) \cdot 20.$$

From the second equation we have

$$\|\mathbf{F}_2\| = 20 \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{2} = 10\sqrt{6}.$$

Using this in the first equation we find

$$\|\mathbf{F}_1\| = \frac{1}{\sqrt{2}} \cdot 10\sqrt{6} + \frac{1}{2} \cdot 20 = 10 + 10\sqrt{3}$$

13.2

PQ2)

The components of $\mathbf{v}$ do not depend on its basepoint, so $\langle 3,2,1\rangle$.

PQ4)

$\overrightarrow{QP} = \langle 2,1,0\rangle$ so (c) is a direction vector but neither of (a) nor (b) are.

PQ6)

True.

4)

(A) and (C) are right-hand ruled, but (B) is not.

34)

$\mathbf{r}(t) = \langle 4,0,8\rangle + t\langle 1,0,1\rangle = \langle 4+t,0,8+t\rangle$.

46)

We have a point on the line: $(4,9,8)$. We need a direction vector. The line is perpendicular to the yz-plane, and so it is parallel to anything perpendicular to that plane, i.e., to the x-axis. Hence, we can use $\langle 1,0,0\rangle$ as a direction vector and then a parametrization is

$$\mathbf{r}(t) = \langle 4,9,8\rangle + t\langle 1,0,0\rangle.$$

52)

If $\mathbf{r}_1$ and $\mathbf{r}_2$ intersect, then there are values of t and s so that

$$\begin{aligned}\langle 2,1,1\rangle + t\langle -4,0,1\rangle &= \langle -4,1,5\rangle + s\langle 2,1,-2\rangle,\\ \langle 2-4t,1,1+t\rangle &= \langle -4+2s,1+s,5-2s\rangle\end{aligned}$$

From the y-coordinate we derive that $s=0$ and so then the first coordinate implies that $\frac{3}{2}=t$, where as the third implies $t=4$, and so there is no solution and so no intercept.

13.3

PQ2)

The angle is obtuse as

$$\mathbf{a}\cdot\mathbf{b} = \|\mathbf{a}\|\,\|\mathbf{b}\|\cos\theta$$

where θ is the angle we are after. So, $\cos\theta < 0$ which implies $\theta \in (\pi/2,\pi)$.

PQ4)

The projection of $\mathbf{v}$ along $\mathbf{v}$ is $\mathbf{v}$ as

$$\mathbf{v}_{||\mathbf{v}} = \left(\frac{\mathbf{v}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v} = \mathbf{v}$$

PQ6)

$\mathbf{e_u}\cdot\mathbf{e_v}$ is correct as

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \mathbf{e_u}\cdot\mathbf{e_v}$$

46)

a)

$$\mathbf{v}\cdot\mathbf{w} = \|\mathbf{v}\|\cdot\|\mathbf{w}\|\cos\theta = 2\cdot 3\cdot\cos(120^\circ) = 6\cdot(-\frac{1}{2}) = -3$$

b)

$$\begin{aligned}\|2\mathbf{v}+\mathbf{w}\| &= \sqrt{(2\mathbf{v}+\mathbf{w})\cdot(2\mathbf{v}+\mathbf{w})}\\ &= \sqrt{4\mathbf{v}\cdot\mathbf{v}+\mathbf{w}\cdot\mathbf{w}+4\mathbf{v}\cdot\mathbf{w}}\\ &= \sqrt{4\cdot 4+9+4\cdot(-3)}\\ &= \sqrt{13}\end{aligned}$$

c)

$$\begin{aligned}\|2\mathbf{v}-3\mathbf{w}\| &= \sqrt{(2\mathbf{v}-3\mathbf{w})\cdot(2\mathbf{v}-3\mathbf{w})}\\ &= \sqrt{4\mathbf{v}\cdot\mathbf{v}+9\mathbf{w}\cdot\mathbf{w}-12\mathbf{v}\cdot\mathbf{w}}\\ &= \sqrt{4\cdot 4+9\cdot 9-12\cdot(-3)}\\ &= \sqrt{133}\end{aligned}$$

74)

$A=(0,0,1), B=(1,0,0)$, and $D=(0,1,0)$ so

$$\overrightarrow{AB} = \langle 1,0,-1\rangle\,,\quad \overrightarrow{AD} = \langle 0,1,-1\rangle$$
$$\left\|\overrightarrow{AB}\right\| = \sqrt{2},\quad \left\|\overrightarrow{AD}\right\| = \sqrt{2}.$$

Therefore,

$$\cos\theta = \frac{\overrightarrow{AB}\cdot\overrightarrow{AD}}{\left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{AD}\right\|} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}.$$

Thus $\theta = 60^\circ$.