HOMEWORK 11 | **SELECTED SOLUTIONS**

Math 1920 | November 8, 2018

17.3

10) SOLUTION: Observe that the field $\mathbf{F} = \langle y, x, z^3 \rangle$ satisfies the cross partials condition everywhere, hence $\mathbf{F}$ is conservative. If $f(x,y,z)$ is a potential function, then it satisfies

$$f(x,y,z) = \int y \mathrm{d}x = xy + f(y,z)$$

$$f(x,y,z) = \int x \mathrm{d}y = xy + g(x,z)$$

$$f(x,y,z) = \int z^3 \mathrm{d}z = \frac{1}{4}z^4 + h(x,y).$$

These three ways of writing f must be equal. The function $f(x,y,z) = xy + \frac{1}{4}z^4 + C$ does the job for any constant C.

15) SOLUTION: This vector field $\mathbf{F}$ satisfies the cross partials conditions everywhere, hence it is conservative. If $f(x,y,z)$ is a potential function, then it satisfies

$$f(x,y,z) = \int (2xy+5)\mathrm{d}x = x^2y + 5x + f(y,z)$$

$$f(x,y,z) = \int (x^2 - 4z)\mathrm{d}y = x^2y - 4yz + g(x,z)$$

$$f(x,y,z) = \int -4y\mathrm{d}z = 4yz + h(x,y).$$

These three ways of writing f must be equal. The function $f(x,y,z) = x^2y + 5x - 4yz + C$ does the job for any constant C.

25) SOLUTION: Work against gravity is calculated with the integral

$$W = -\int_{\mathcal{C}} m\mathbf{F} \cdot \mathrm{d}\mathbf{s} = 1000 \int_{\mathcal{C}} \nabla V \cdot \mathrm{d}\mathbf{s} = 1000(V(r_2) - V(r_1)).$$

Since r_1 and r_2 are measured from the center of the earth,

$$r_1 = 4 \times 10^6 + 6.4 \times 10^6 = 10.4 \times 10^6 \text{ meters}$$

$$r_2 = 6 \times 10^6 + 6.4 \times 10^6 = 12.4 \times 10^6 \text{ meters}$$

$$V(r) = -\frac{k}{r} \Rightarrow W = -\frac{1000k}{10^6}\left(\frac{1}{12.4} - \frac{1}{10.4}\right) \approx 6.2 \times 10^9 \text{ J}.$$

28a) SOLUTION: If $f(x,y,z)$ is a potential function for $\mathbf{F}$, then

$$f(x,y,z) = 400 \int \left(\frac{x}{x^2+z^2}\right) \mathrm{d}x = 200 \ln(x^2+z^2) + f(y,z)$$

$$f(x,y,z) = \int 0 \mathrm{d}y = g(x,z)$$

$$f(x,y,z) = 400 \int \left(\frac{z}{x^2+z^2}\right) \mathrm{d}z = 200 \ln(x^2+z^2) + h(x,y).$$

The function $f(x,y,z) = 200\ln(x^2+z^2)$ is a potential function for $\mathbf{F}$. Let

$$V(x,y,z) = -f(x,y,z) = -200\ln(x^2+z^2);$$

Then $\mathbf{F} = -\nabla V$.