Math 1920, Prelim 2
November 10, 2015, 7:30 PM to 9:00 PM

There are **6 questions** for a total of 100 points. You are NOT allowed to use calculators, cell phones, electronic devices, the text or any other book or notes. **Show all work!**
Writing clearly and legibly will improve your chances of receiving the maximum credit that your solution deserves. Please label the questions in your answer booklet clearly.

1. (15 points) Consider the function
$$f(x, y) = \frac{4}{3}x^3 + y^2x - x.$$
   (a) Find the critical points for this function.
   (b) Determine whether each of these is a local maximum, local minimum, or saddle point.
   (c) Does $f(x, y)$ have a global maximum or global minimum? Why or why not?

2. (15 points) Use the method of Lagrange multipliers to find the point on the ellipsoid
$$\frac{x^2}{3} + \frac{y^2}{2} + \frac{z^2}{4} = 1$$
that maximizes the sum $x + y + z$.

3. (15 points) Consider the following iterated triple integral.
$$\int_0^2 \int_0^{y/2} \int_0^{2-y} xyz \, dz \, dx \, dy.$$
   (a) Sketch the region of integration, and indicate the equations of the four bounding surfaces on the sketch.
   (b) Write this integral in the order $dy \, dz \, dx$. Do not evaluate this integral.

4. (15 points) Consider the hyperboloid given by
$$x^2 + 2y^2 - 3z^2 = 1.$$
Find all the points $P = (a, b, c)$ on the hyperboloid such that the tangent plane at P is parallel to the plane
$$3x + 2y + z = 0.$$

5. (20 points) Let D be the region in space where $x > 0$, $y > 0$, and $z > 0$. Consider the vector field
$$\mathbf{F} = \left\langle \frac{1}{x^2y}, \; 1 + \frac{1}{xy^2}, \; \frac{1}{z} \right\rangle.$$
   (a) Is D simply connected?
   (b) Find curl $\mathbf{F}$.
   (c) Find a potential function for $\mathbf{F}$, if one exists. If one doesn't exist, explain why.

6. (20 points) Little Johnny has built a teleporter. Teleported objects materialize inside the half-ball
$$x^2 + y^2 + z^2 \leq 1, \quad z \geq 0.$$
The location of entry is random, with probability density $p(x, y, z) = kz^2$ (Note: $p(x, y, z) = 0$ outside the half-ball).
   (a) What is the value of k which makes $p(x, y, z)$ a valid probability density function?
   (b) What is the probability that the object materializes in the region above $z = \sqrt{x^2 + y^2}$ with $y \geq 0$?