



Math 1920

Prelim 2

Name _____

7 November 2017

7:30–9:00 pm

PLACE AN X IN THE BOX BY YOUR DISCUSSION SECTION NUMBER

<input type="checkbox"/> 201 Aleksandra Niepla MW 7:30–8:20P	<input type="checkbox"/> 214 Shrinidhi Pandurangi TR 9:05–9:55A
<input type="checkbox"/> 202 Aleksandra Niepla MW 8:35–9:25P	<input type="checkbox"/> 215 Andres Fernandez TR 9:05–9:55A
<input type="checkbox"/> 203 Andres Fernandez TR 8:00–8:50A	<input type="checkbox"/> 216 Dylan Peifer TR 9:05–9:55A
<input type="checkbox"/> 204 Dylan Peifer TR 8:00–8:50A	<input type="checkbox"/> 217 Feng Liang TR 12:20–1:10P
<input type="checkbox"/> 205 Thomas Reeves TR 8:00–8:50A	<input type="checkbox"/> 218 Max Jenquin TR 12:20–1:10P
<input type="checkbox"/> 206 Bram Wallace TR 8:00–8:50A	<input type="checkbox"/> 219 Feng Liang TR 1:25–2:15P
<input type="checkbox"/> 207 Shrinidhi Pandurangi TR 8:00–8:50A	<input type="checkbox"/> 220 Ryan McDermott TR 1:25–2:15P
<input type="checkbox"/> 209 Bill Wu TR 8:00–8:50A	<input type="checkbox"/> 221 Itamar Oliveira MW 7:30–8:20P
<input type="checkbox"/> 210 Thomas Reeves TR 9:05–9:55A	<input type="checkbox"/> 222 Itamar Oliveira MW 8:35–9:25P
<input type="checkbox"/> 211 Bram Wallace TR 9:05–9:55A	<input type="checkbox"/> 223 Max Jenquin TR 1:25–2:15P
<input type="checkbox"/> 213 Bill Wu TR 9:05–9:55A	<input type="checkbox"/> 224 Ryan McDermott TR 2:30–3:20P

INSTRUCTIONS—PLEASE READ NOW

- Write your name and check the box with your discussion section number *right now*.
- There are 6 problems and this booklet has 7 sheets. You may use the back of each sheet as scratch paper.
- You have 90 minutes to complete the exam. You may leave early, but if you finish within the last 15 minutes, please remain in your seat.
- Show your work and simplify your answers. To receive full credit, your answers must be neatly written and logically organized.
- You are allowed a one-sided letter size sheet of notes. No books or electronic devices or any other resources are allowed.
- Academic integrity is expected of all Cornell University students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below to indicate that you have read and agree to these instructions.

Signature of Student

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1. _____ / 15

2. _____ / 15

3. _____ / 15

4. _____ / 15

5. _____ / 20

6. _____ / 20

Total _____ / 100

1 (15 points). Find the global minima and maxima of the function $f(x, y, z) = -x + 2y + z$ over the solid ellipsoid given by $x^2 + y^2 + \frac{1}{4}z^2 \leq 1$.

2 (15 points). Let \mathcal{W} be the solid enclosed between the surface $z = \sqrt{x^2 + y^2}$ and the plane $z = h$, where h is a positive constant.

- (a) Find the volume of \mathcal{W} .
- (b) Find the centroid of \mathcal{W} . (Recall that the centroid is the center of mass with respect to a constant density function equal to 1.) You may use the symmetries of \mathcal{W} to shorten your calculations.

(The answers will depend on h .)

3 (15 points). Find the work done by the force $\mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$ along the path $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ for $0 \leq t \leq T$. (The answer will depend on T .)

4 (15 points). Let a be a constant and let \mathbf{F} be the vector field

$$\mathbf{F}(x, y) = \left\langle x \ln y + ye^x, \frac{1}{2}x^2y^a + e^x + 1 \right\rangle.$$

The domain of \mathbf{F} is the planar region \mathcal{D} defined by $y > 0$.

- (a) Is \mathcal{D} simply connected?
- (b) For what value of a is \mathbf{F} conservative?
- (c) For the value of a you found in part (b), find a potential of \mathbf{F} .

5 (20 points). Let $f(x, y) = x^3 + y^3 - 3xy$.

- (a) Find all critical points of f in the plane \mathbf{R}^2 .
- (b) For each critical point state whether it is a local minimum, local maximum, or saddle point.
- (c) Does f have global extrema? Explain your answer.

6 (20 points). Let \mathcal{W} be the solid enclosed by the two spheres $x^2 + y^2 + z^2 = R^2$ and $x^2 + y^2 + (z - R)^2 = R^2$, where R is a positive constant.

- (a) Sketch the solid \mathcal{W} . Include sufficient detail, such as coordinate axes and some indication of scale.
- (b) Set up the integral $\iiint_{\mathcal{W}} xyz \, dV$ in cylindrical coordinates with the variables in the order $dz \, dr \, d\theta$. Write the answer as an integral with explicit limits of integration. Do not attempt to evaluate the integral.
- (c) In the integral of part (b) change the order of integration to $dr \, dz \, d\theta$. Do not attempt to evaluate the answer.