MATH 1920 - Fall 2018 - Prelim 1 Practice 1

1. Show USING CALCULUS (that is no physics arguments) that if a particle moves along a path given by a twice differentiable position function $\mathbf{r}(t)$ at a constant speed, then its velocity and acceleration vectors must be perpendicular to each other.

2. Water in a river flows with constant velocity $v = \langle 2, 1, -1\rangle$ cm/s. A circular screen of radius 9cm is placed in the river.

   (a) If the screen is placed so that it lies in the plane given by $3x + 4y + 5z = 6$, find the volumetric flow rate of the water across the screen.

   (b) What direction of the screen (i.e., what direction of the normal to the plane in which the screen lies) would make the flow rate across the screen as large as possible?

   (c) At time $t = 0$ a small bug lands in the river at point $(1, 2, 3)$, and moves along with the flow of water. Give a parametric equation for the bug's position $t$ seconds later if it has the same velocity as the water.

   (d) If the bug continues down the river could it eventually hit the screen if it lies in the plane $3x + 4y + 5z = 6$? Justify your answer.

3. A syzygy in astronomy is a straight line configuration of three heavenly bodies (such as a lunar or solar eclipse involving the Earth, Moon, and Sun). Consider three points whose position at time $-\infty < t < \infty$ is given by the vector functions

$$\mathbf{r}_1(t) = 4\mathbf{k} + t\mathbf{k}, \qquad \mathbf{r}_2(t) = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + t\mathbf{j}, \qquad \mathbf{r}_3(t) = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + t\mathbf{i}.$$

   Find the value of $t$ when the three points form a syzygy.

4. A fighter plane, which can shoot a laser beam straight ahead, travels along the path $\mathbf{r}(t) = \langle 2 + \cos(t), \sin(t)\rangle$. Show that there is precisely one time $t$ for $0 \le t \le 2\pi$ at which the pilot can shoot a target located at the point $(4, 0)$. Calculate this time $t$.

5. Calculate the limits or show that they do not exist. Remember to justify your answers.

   (a)
$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2 + y^2}$$

   (b)
$$\lim_{(x,y)\to(0,0)} \frac{x^2\sqrt{|y|}}{x^2 + y^2}$$

6. The velocity of a certain particle at time $t$ is given by

$$\mathbf{v}(t) = \langle t, \pi\cos(\pi t), 1\rangle.$$

   At time 0 the particle is at the point $(0, 1, -1)$.

(a) Find the position $\mathbf{r}(t)$ of the particle at time $t$.

(b) Find the speed of the particle at time $t$.

(c) Set up an integral for the distance traveled by the particle between the points $(0, 1, -1)$ and $(8, 1, 3)$. Do not attempt to evaluate the integral.

7. Find a parametrization for the curve which is the intersection of the cylinder $y^2+z^2 = 4$ and the surface $x = y^2z$. Give bounds on the parameter so that the curve is traced exactly once.