MATH 1920 - Fall 2017 - Prelim 1 Practice 2

1. Consider the vectors $\mathbf{v} = \mathbf{i} + 2\mathbf{j} + a\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

   (a) Find all values of the number a (if any) such that $\mathbf{v}$ is perpendicular to $\mathbf{w}$.

   (b) Find all value of the number a (if any) such that the area of the parallelogram determined by $\mathbf{v}$ and $\mathbf{w}$ is equal to $\sqrt{6}$.

2. Find an equation for the plane through the origin perpendicular to the plane $2x + 2y + z = 1$ and perpendicular to the vector $\mathbf{v} = \langle 1, 1, -4 \rangle$.

3. Consider the function $g(x, y) = \sqrt{y^2 - x^2}$.

   (a) Sketch the level curves $g(x, y) = c$ for $c = 0, 1, 2$.

   (b) What is the domain of g?

4. The wave equation, where a^2 is constant, is given by

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}.$$

   It describes the motion of a waveform such as fluid, sound, or light. Suppose $u(x, t)$ represents the displacement of a vibrating guitar string at time t at a distance x from one end of the string. If $u(x, t) = \sin(x - at)$, show that it satisfies the wave equation.

5. Calculate each of the following limits or show it does not exist.

   (a)
$$\lim_{\substack{(x,y)\to(4,3) \\ x \neq y+1}} \frac{\sqrt{x} - \sqrt{y+1}}{x - y - 1}$$

   (b)
$$\lim_{(x,y)\to(0,)} \frac{y}{\sqrt{x^2 + y^2}}$$

6. Find an equation for the tangent plane to the surface

$$z = g(x, y) = 1 + \frac{4x^2}{y} + \ln(x^2 + y^2 - 4)$$

   at the point $(1, 2, 3)$.