MATH 1920 - Fall 2018 - Prelim 1 Practice 3

1. Consider the function $z = f(x, y) = \frac{x^2}{1+y^2}$.

   (a) Find an equation of its tangent plane at $(4, 1)$ in the form $ax + by + cz = d$.

   (b) Use this equation or one equivalent to it to obtain a linear approximation to the function at $(4.01, 0.98)$.

2. Consider the four points $A = (1, 1, 1)$, $B = (1, -2, 1)$, $C = (0, 3, 2)$, and $D = (1, 2, a)$. Let P be the plane containing A, B, and C.

   (a) Find an equation of the plane P.

   (b) For which value(s) of a is the point D on this plane?

   (c) Find the area of the triangle ABC.

   (d) When the point D is not on the plane, find the distance from the point D to the plane P. Your answer should involve a.

3. Show that $\rho = 4\cos(\phi)$ is the equation in spherical coordinates of a sphere with its center on the z-axis. What is its radius and where is its center?

4. Let $z = f(x, y) = \sqrt{3 - x^2 - y^2}$ and $z = g(x, y) = (x^2 + y^2)/2$ be two surfaces in space. Consider the curve which is the intersection of these surfaces.

   (a) Find a parametrization $\mathbf{r}_1(t)$ of this curve (including bounds on the parameter).

   (b) Find a parametrization $\mathbf{r}_2(t)$ for the tangent line to this curve at $(1, 1, 1)$.

5. Sketch and describe the set of points that have spherical coordinates $\rho = 2$, $0 \leq \theta \leq 2\pi$, and $0 \leq \phi \leq \frac{\pi}{2}$.

6. Find an equation in cylindrical coordinates for the sphere of radius 2 centered at the origin.

7. Let $f(x, y) = xy\sqrt{x^2 + y^2}$.

   (a) Compute $f_x(x, y)$ and $f_y(x, y)$ for $(x, y) \neq (0, 0)$.

   (b) Using the original limit definition of partial derivatives, verify that

   $$f_x(0, 0) = f_y(0, 0) = 0.$$

   (c) Determine whether the limit

   $$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{\sqrt{x^2 + y^2}}$$

   exists, and if so, find its limit. Show your reasoning.

   (d) Are f_x and f_y continuous at $(0, 0)$? What can you say about the differentiability of f at $(0, 0)$?