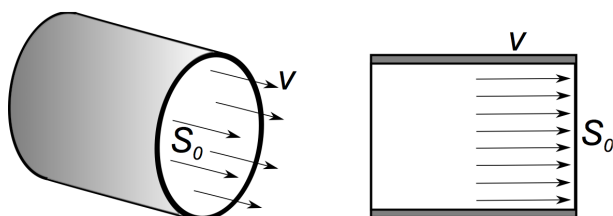


Math 1920 Workshop 1: Flux

This workshop is an application of the dot product: flux. Consider a fluid flowing through a region in space. Imagine a slice or a cross-section of the region. Then the volume of fluid that flows across the cross-section per unit time is called the flux.

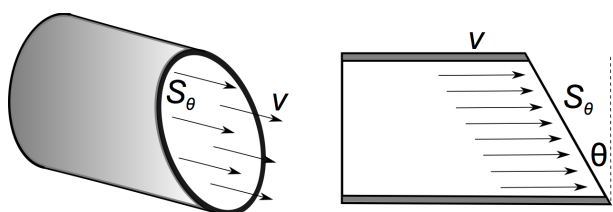
Problem 1 – Flux through a pipe

- a) Fluid is flowing down a pipe at velocity $\vec{v} = 5$ m/sec. The cross sectional area S_0 of the pipe is 3 m^2 . What is the volume of fluid flowing out of the pipe each second? This flow rate is called the volumetric flux of the fluid velocity.



- b) Suppose the pipe is cut on the diagonal as shown in the figure below. Now it has a new cross-sectional area S_θ at an angle θ . Note that $S_\theta = \frac{S}{\cos\theta}$. Briefly state why this makes sense.

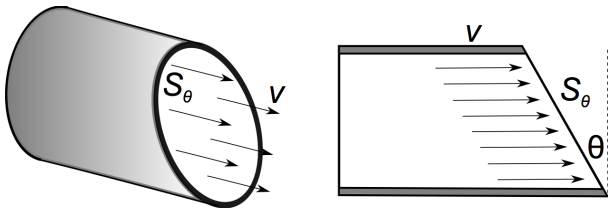
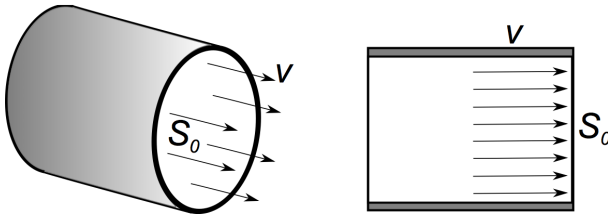
Is the flux out of the pipe greater than, less than or equal to the one in part a)? Explain your answer.



- c) Let us now treat area as a vector. Consider a generic flat surface with constant area S and *unit vector* (a vector of length 1) \vec{n} , normal to the surface pointing ‘outward’. We can define the area vector as $\vec{S} = S\vec{n}$. The magnitude of \vec{S} is the area S , and its direction is normal to the area.

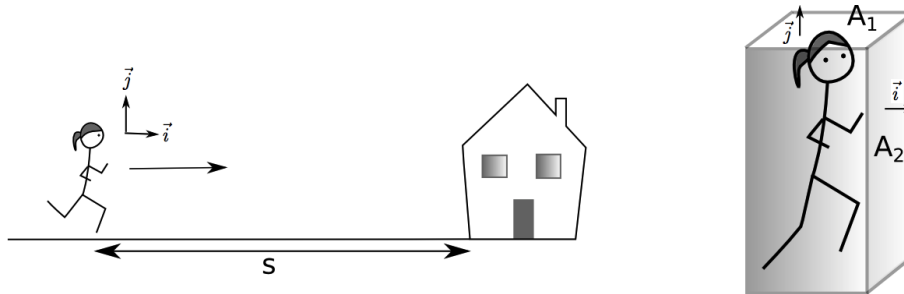
If we have a constant velocity vector defined over our surface, the flux is the dot product of area and velocity vectors, i.e. $\text{flux} = \vec{v} \cdot \vec{S}$.

Use this definition of flux to redo part **a)** and **b)**. Draw the normals to S_0 and S_θ and calculate the flux as $\vec{v} \cdot \vec{S} = S(\vec{v} \cdot \vec{n})$.

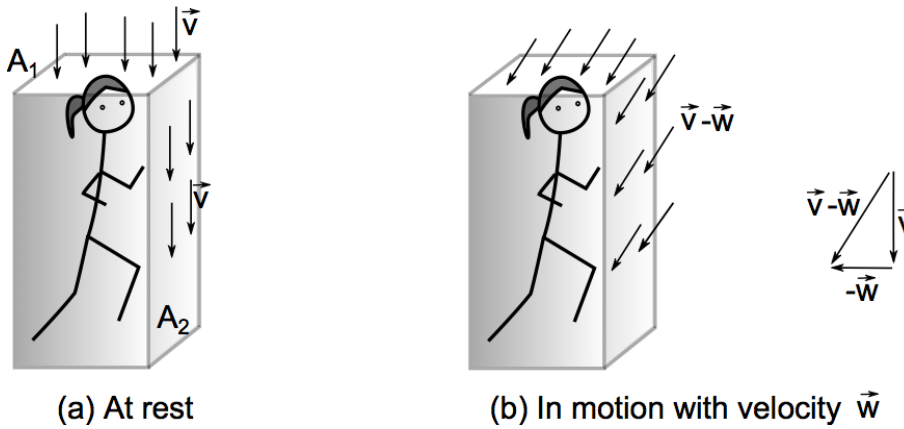


Problem 2 – A fun question to ponder

Imagine that you are on your way home when suddenly it rains - and you don't have an umbrella! Home is a distance s away when the rain starts falling. For simplicity let's assume that you are modeled as a box, as shown below.



Let \vec{i} be a unit vector perpendicular to area A_2 and \vec{j} a unit vector perpendicular to area A_1 . Figures (a) and (b) below show how the rain appears to fall when you are stationary and when you are moving forward with velocity \vec{w} . We assume the rain is evenly distributed and falling straight down with velocity \vec{v} . The velocity of the rain relative to your body is $\vec{v} - \vec{w}$.



- a) Write down the area vectors for the surfaces A_1 and A_2 using \vec{i} and \vec{j} as direction vectors. Write \vec{v} and $\vec{v} - \vec{w}$ in terms of \vec{i} and \vec{j} . Recall that \vec{v} has length $\|\vec{v}\|$ and the surfaces A_1 and A_2 have area $\|A_1\|$ and $\|A_2\|$ respectively.

b) The amount of water hitting a surface **per unit time** is represented by the flux through the surface. Calculate the amount of water hitting A_1 per unit time. Calculate the amount of water hitting A_2 per unit time.

c) How long will it take you to travel the distance s to your house when you are traveling with velocity \vec{w} ? Write an equation that calculates how much water will hit you as you travel the distance s to your house.

d) If you want to minimize how wet you get, is it better to walk or run through the rain?

In the Future:

If the velocity vector is constant over the area, the flux is the dot product of area and velocity vectors: $\text{flux} = \vec{v} \cdot \vec{S}$.

If the area is curved in 3D, and/or the vector field is not constant over the area flux is a 'surface integral': $\int_S \vec{v} \cdot d\vec{S}$ of the vector field \vec{v} through the surface S in space. It is the sum (integration) of the 'small fluxes' $(\vec{v} \cdot d\vec{S})$ through small area vectors $d\vec{S}$ on the surface S . We'll see surface integrals in the future.