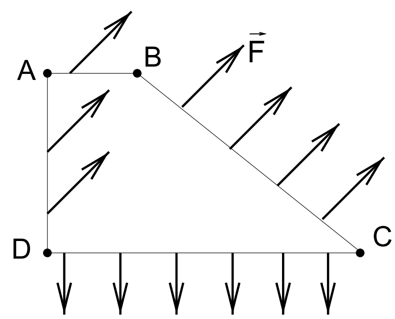
Math 1920 Workshop 2: Work

By the physics and engineering definition, you do *work* by exerting a force on an object as it undergoes a displacement from one place to another. If the force $\vec{F}$ is constant and acts in the same direction as the displacement $\vec{s}$, the work done is $W = |\vec{F}||\vec{s}|$. If the constant force is not in the direction of the displacement, the work done is $W = \vec{F} \cdot \vec{s}$. Note that the first formula is a special case of the second.

Problem 1 – A simple example
The figure below shows the path traced by an object as a force is exerted on it. The magnitude of the force $\vec{F}$ is constant on each of the segments of the path, although the direction varies.



a) Can you determine from the information given whether the work done by $\vec{F}$ in moving an object on the path ADC (from point A to point C via point D) is positive, negative, or zero? Explain.

b) A force is called *conservative* if the work done in moving an object from one point to another does not depend on the path taken, but instead depends only on the endpoints. Can you determine from the information given whether $\vec{F}$ is conservative? Explain.

Problem 2 – Gravity

If the force is **not** constant along the path AB, the work done is the summation of work done along small displacements $d\vec{s}$ on the path, and is given by the 'line integral': $\int_A^B \vec{F} \cdot d\vec{s}$. We'll learn how to calculate line integrals in the future.

The figure below is a picture of the Earth and its gravitational field. Suppose a spaceship travels from point A to point E on the figure below. The force on the spaceship is given by

$$\vec{F}(x,y) = -GMm \left\langle \frac{x}{(x^2+y^2)^{3/2}}, \frac{y}{(x^2+y^2)^{3/2}} \right\rangle$$

where G is the gravitational constant, M is the mass of the Earth, and m is the mass of the spaceship. Recall that $x^2 + y^2 = R^2$ where R is the distance from the spaceship to the center of the Earth.

Can you tell if gravity would do a different amount of work if the spaceship travels along the path ACE than if it travels along the path $ABDE$? A good strategy would be to first determine which pieces of the path will give you a zero line integral (think about where the dot product would be zero) and then examine what is left. Do you think $\vec{F}(x,y)$ is conservative?