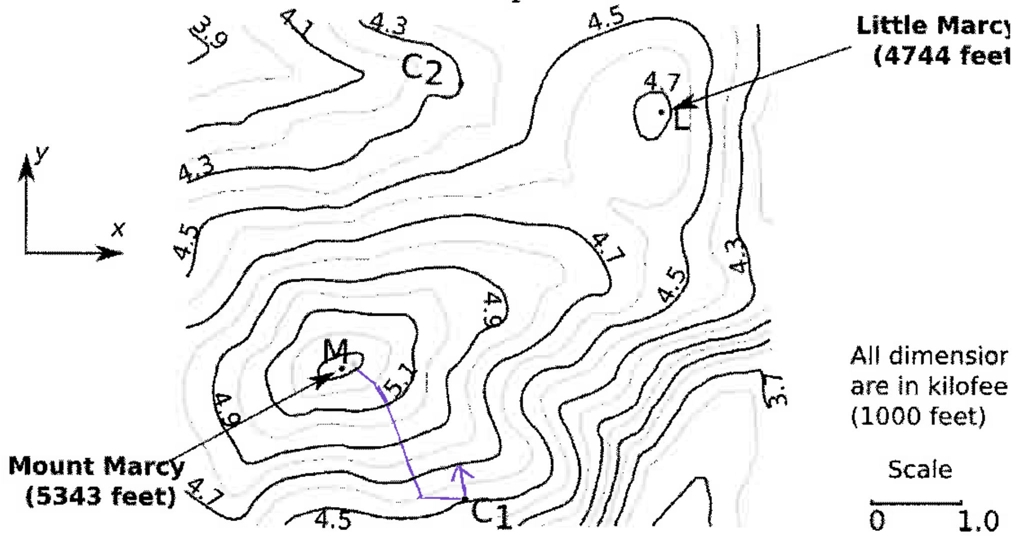
Math 1920, Workshop 3 : Level Curves of a Mountain: Solutions

Map 1

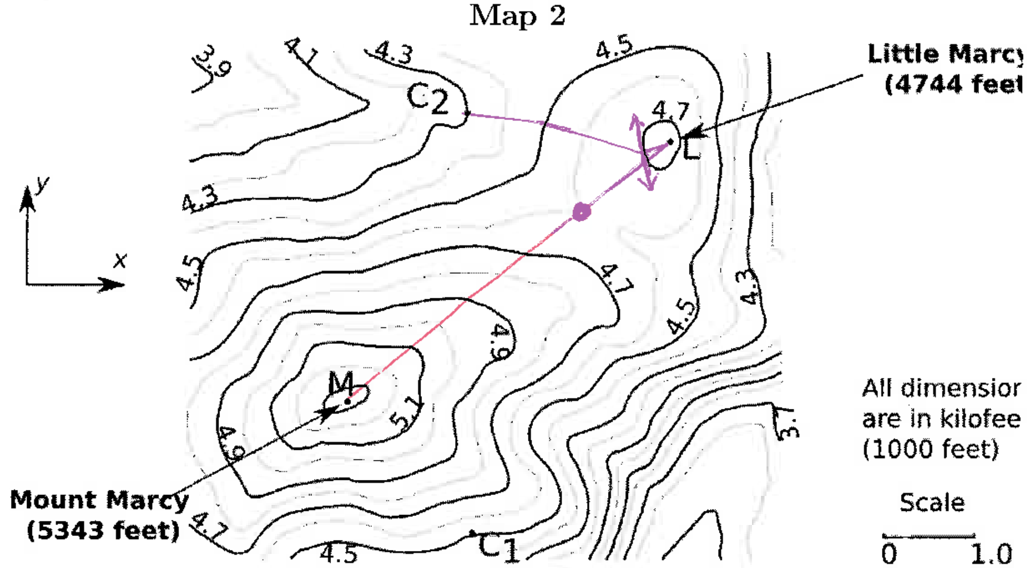


a) See Map 1. The path of steepest ascent is perpendicular to the contours.

b) See Map 1. This is one of many possible solutions. We start off initially going up slowly and then once we reach the first contour we try to stay perpendicular to them as we go up the mountain.

c) See Map 2. Note that we end up at L, the peak of Little Marcy.

Map 2



d) See Map 2. Note that to stay at the same altitude we must walk tangent to the contour lines, and we can travel either clockwise or counter-clockwise.

e) See Map 2. The lowest point is roughly at height 4600 feet. We could use Lagrange multipliers to find the exact minimum of $f(x, y)$ using the constraint that it stays on line LM.

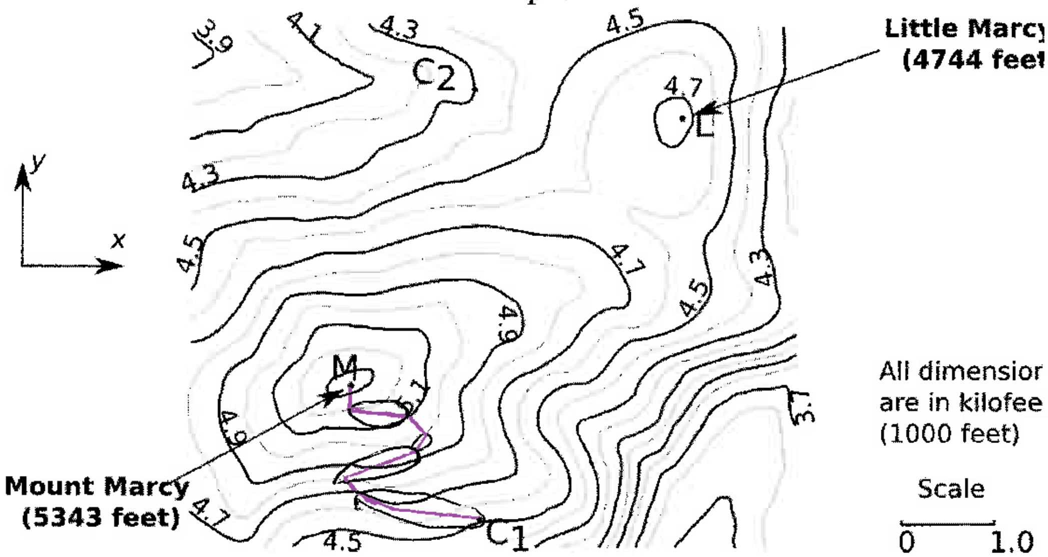
f) Let $\mathbf{u}(a,b)$ be a unit vector which is tangent to your path up the mountain at point $P=(a,b)$. The statement in the problem says that generally the directional derivative in the direction of $\mathbf{u}$ must be less than or equal to .15 i.e.

$$D_{\mathbf{u}}f(P) \leq .15$$

except for short distances along the path.

g) The path we drew in part b does not satisfy these conditions. See Map 3 for one which does. Again there are a number of possible paths which can work. .

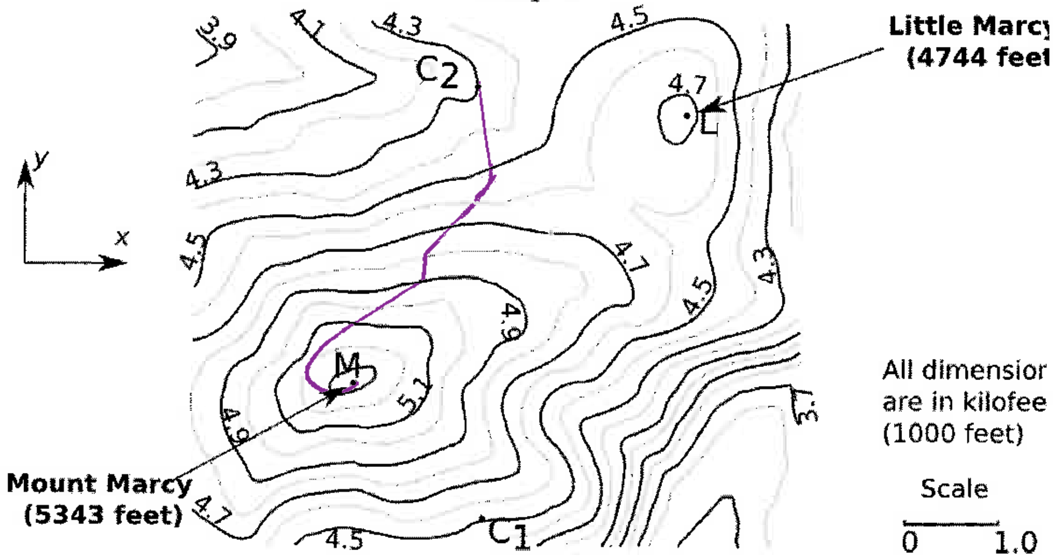
Map 3



h) Outsloping is required where the trail is not rising very steeply. When you are perpendicular or close to perpendicular to the contour lines most of the slope is going upward, whereas if the angle between your path and the contours is further from 90° then most of the slope will be either on the left or the right side of the path. See Map 3.

i) The recommended maximum grade that can accommodate both snowmobiles and ATVs is 25%. See Map 4.

Map 4



TAKE HOME POINTS

- For a function $z = F(x, y)$, the gradient $\nabla F = \langle F_x, F_y \rangle$ is in the direction perpendicular to the level set/contour in xy-plane at the point x, y.
- Gradient is the direction of steepest slope for a function and its magnitude is that slope.
- Slope along the level sets or contours is zero.
- Choosing a path of steepest ascent may NOT lead you to the global maximum! This fact is important in optimization problems and algorithms.