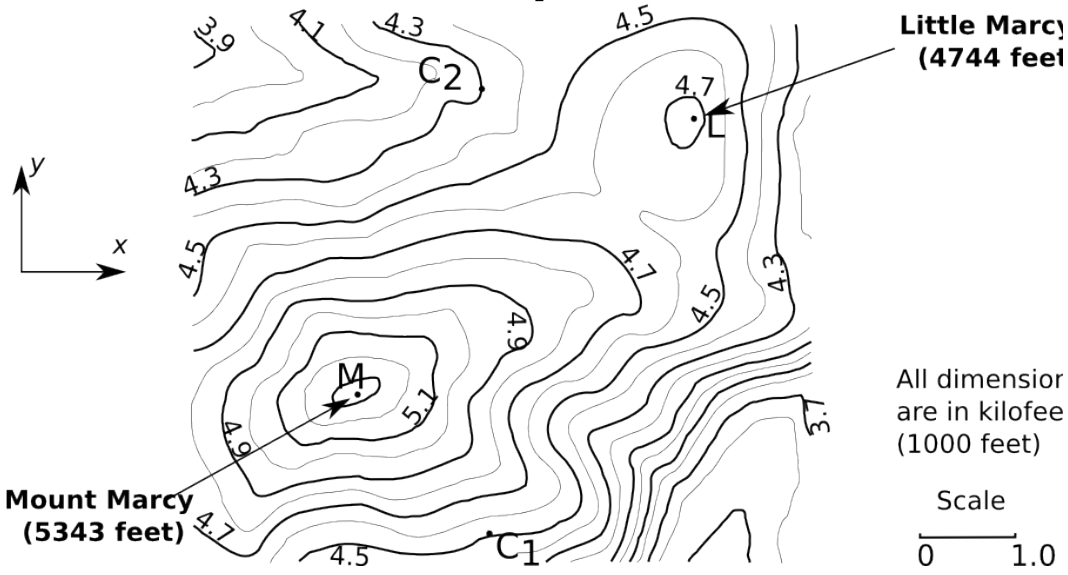
Math 1920, Workshop 3: Level Curves of a Mountain

Mount Marcy in the Adirondack High Peaks is the highest point in New York State. The park rangers in the area are looking to expand their trail systems and hires your team to consult. Your team consists of a land planner who specializes in building hiking trails, and a number of experienced hikers who are comfortable hiking off trail. There are already two hiking trails to the top of Mount Marcy; one approaching the peak from the south west and the second approaching from the north east. The park rangers would like to add a third hiking trail with steeper slopes coming from the south east, and a fourth trail which can accommodate ranger vehicles.

For our purposes, we can model Mt. Marcy and the nearby Mt. Little Marcy as a graph of a function of two variables, $z = f(x, y)$. The level sets of this function, i.e. the curves in xy-plane where the mountain height z is constant, are available to you as a map below. The level sets (a.k.a. contours) are spaced by 0.1 kilofeet (100 ft) in their z values.

Map 1

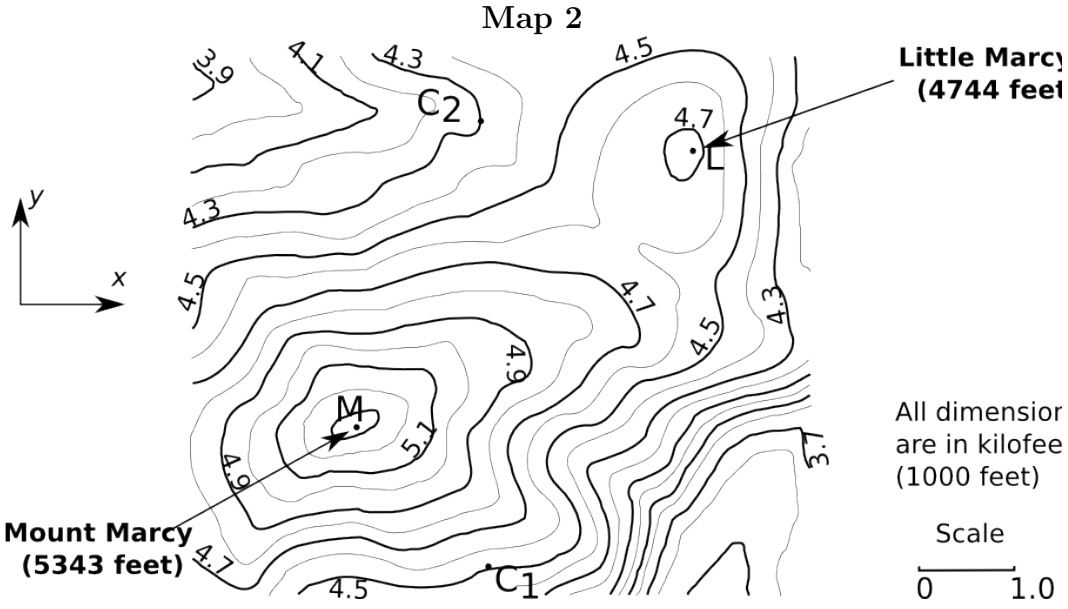


Your team splits into two groups, one which hikes up from pt C_1 in the south, and one which hikes up from pt C_2 in the north.

a) The first group starts at the camping location C_1. One of you suggests the direction of steepest ascent for "a strong start". Draw a vector in that direction, at point C_1 on Map 1 above. Justify.

b) The group decides eventually to take a slightly slower start to warm up, at least until the first contour. Draw a path on Map 1 which would take a slow start, then climb in the steepest direction after the first contour. There are multiple ways to do this!

c) The second group starts at camping site C_2. You decide to start climbing from there, *following the direction of steepest ascent*. Trace your path on Map 2.



d) As you hit the altitude of 4700 feet, the views are great! The group decides to "loop around for a better view", staying at approximately constant altitude so that you can walk more easily. Assuming there's a suitable ledge to walk along, what vector direction(s) could you walk and show it(them) on Map 2? Justify.

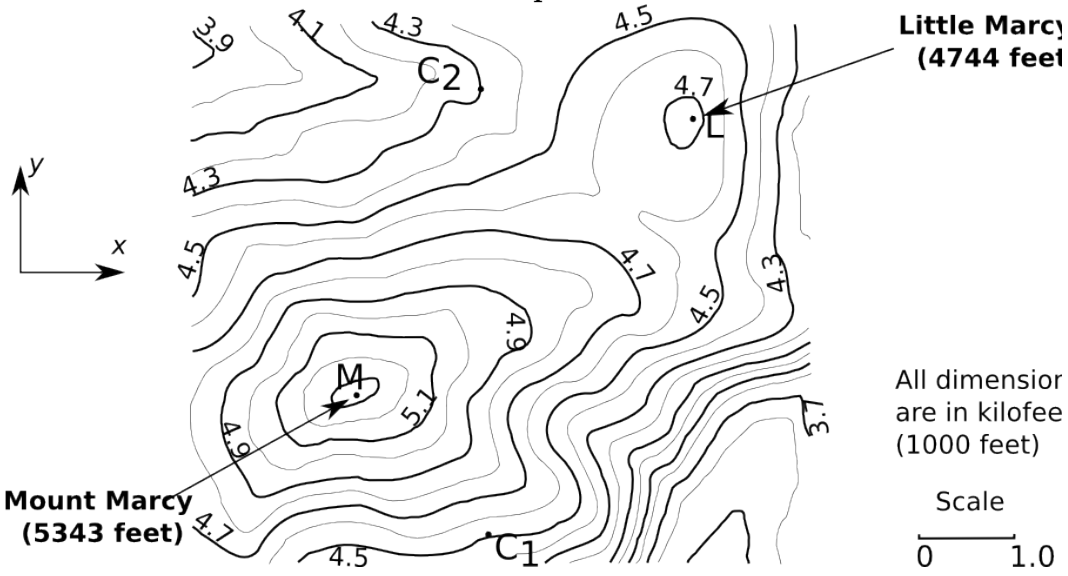
e) The first group finally reached the top of Mt. Marcy (M). The second group eventually reached the top of Little Marcy (L) and now they are heading towards M, by taking a straight path joining the two peaks. Draw the line ML on Map 2. Mark the lowest point on ML. Roughly what height is it at? What method can you use to find the exact height of the lowest point?

Now that you have the lay of the land you need to come up with ideas for the trails the rangers asked you for. One thing to consider while planning these trails is that you would like to minimize erosion. The two main considerations for erosion control are water drainage and the grade(slope) of the trail. We will focus on the grade of the trail today. Note that grade is usually given as a percentage so if the grade is $n\%$, that means the upward slope of the trail is $n/100$.

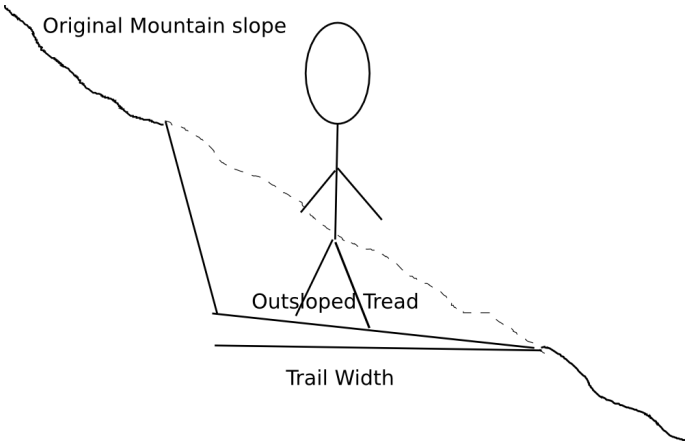
f) The NYS Office of Parks, Recreation and Historical Planning recommends a maximum of 15% grade for a hiking trail in order to minimize erosion, but says that much higher grades are acceptable for short periods of time. Interpret this as a statement about directional derivatives along your path.

g) If we allow much larger grades, up to 45%, for three distinct changes in elevation of 100 feet, does the path you drew in part (b) satisfy these conditions? If not, draw a path from C_1 to M one that does on Map 3.

Map 3



h) When creating a trail you will likely come to a surface where either the left or right side of the trail is significantly higher than the other. In these situations you will need to cut into the mountain on the higher side of the trail to level it out. In order to maintain water drainage down the mountain we usually leave a slight downhill slope to allow the water to continue to travel downhill, a process called outsloping. Circle the portions of your trail where leveling the tread and outsloping will most likely be required.



i) The park rangers want to be able to travel from C_2 to M in both all-terrain vehicles(ATVs) and snowmobiles. The NYS Office of Parks, Recreation and Historical Planning recommends a maximum of 25% grade for snowmobiles and 30% grade for ATVs. However both can handle up to 40% grades for changes in altitude less than 100 feet. What is the recommended maximum grade that can accommodate both snowmobiles and ATVs? Draw in a trail on Map 4 that mostly satisfies this condition, allowing no more than one instance of a grade above the recommended maximum.

Map 4