# Math 1920 Workshop 6 : Gauss's Law

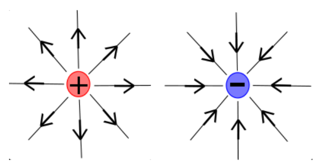
Charges generate electric fields. The electric field, often represented by $\vec{E}$, can be shown as a set of field lines emanating from or entering into the charges, depending on their signs. Gauss's law describes the relationship between the charges and the electric field generated by them – conveniently formulated as an integral over a closed surface $S$.

**Problem 1 – Gauss's law**

Experimentally, we know the following about a point charge (a.k.a. Coulomb's law):

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{e}_r$$

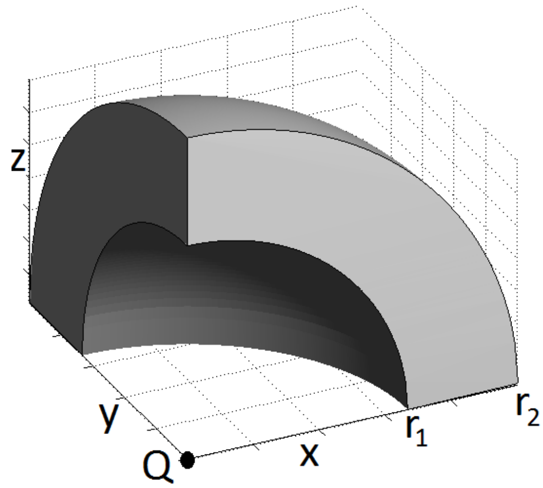
where $Q$ is the charge (positive or negative), $r = ||\vec{r}||$ is the radial distance from the charge's location, $\hat{e}_r = \frac{\vec{r}}{||\vec{r}||}$ is the unit vector in the radial direction, and $\epsilon_0$ is a constant. The field lines showing the direction of $\vec{E}$, are shown below:




**a)** With this information in mind, and using what you know about surface integrals, consider a sphere $S$ of radius $r$, centered on a positive charge of magnitude $Q$. Draw the sphere and the electric field vectors on its surface. Find the surface integral $\oiint_S \vec{E} \cdot d\vec{S}$. This is the flux of electric field outward through the surface $S$.

**b)** Repeat part **a)** if the charge was negative.

**c)** Consider the closed surface $S$ shown below. It is the boundary of the region in the first octant contained between two spheres of radii $r_1$ and $r_2$ centered on the charge $Q$ at the origin. Draw the normal vectors on $S$ (pointing outwards). Find $\oiint_S \vec{E} \cdot d\vec{S}$. Does it matter if $Q$ is a positive or a negative charge?

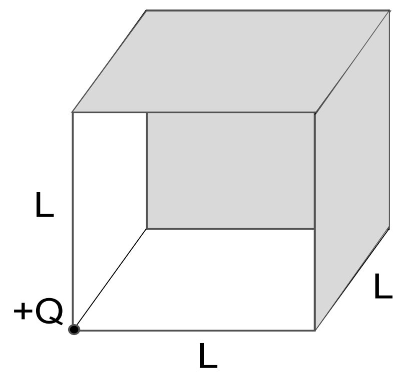



**d)** Gauss's law relates the flux of $\vec{E}$ through a closed surface $S$ to the amount of charge inside it. Based on parts **a)**, **b)** and **c)**, can you make an educated guess as to what Gauss's law says?

$$\oiint_S \vec{E} \cdot d\vec{S} =$$

Even though we guessed Gauss's law for point charges and by integrating over simple surfaces, it is actually valid for any closed surface with any charge distribution.

**e)** A positive charge $Q$ sits at the corner of a square box with sides of length $L$, as shown. Calculate the flux of the point charge's electric field $\vec{E}$ through the **shaded sides**.




*Hint*: The charge $Q$ is not enclosed by the box, it is at the corner, and the shaded sides don't form a closed surface. Hence you can't use Gauss's Law directly. You'll have to think about symmetry.

**Problem 2 – Conductors**

A conductor allows for a free motion of charges inside and on it. Hence if you drop some charge on a conductor, the charge redistributes until there is no electric field inside the conductor to move charges around. Given this information and Gauss's law, construct an argument to show that the charges redistribute such that they always reside on the outer surface of the conductor.

This idea is employed in a Faraday's cage, which is basically a metal container. You can remain safe inside it unaffected by external electric fields or charges, e.g. lightening.