Math 2130   Name (Print):   _____

Spring 2014

Final Exam   Discussion: _____

5/19/14

Time Limit: 150 Minutes   TA: _____

This exam contains 11 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use any electronics on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Do not simplify:** for example, $2(0.7) + \cos(\pi/5)$ is a good number.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	20	
3	35	
4	15	
5	25	
6	15	
7	25	
Total:	150	

1. The figure shows a rectangular box which measures 2 by 1 by h units, where h denotes the height in the z direction. The origin of coordinates is at the midpoint of one low edge, and the two vectors, $\vec{v}$ and $\vec{w}$, go from the origin to the high corners on the other side.
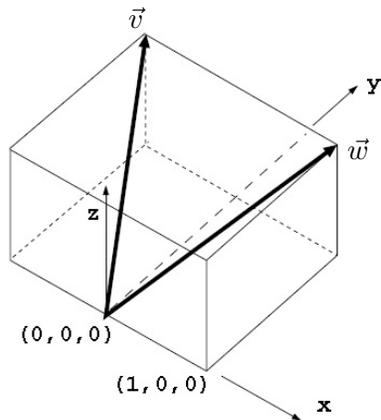
(a) (5 points) Figure out the components of the vectors $\vec{v}$ and $\vec{w}$.

(b) (5 points) Calculate the area between the two vectors in terms of h.

(c) (5 points) Find the height h so that the angle between the two vectors is 60 degrees. (*Hint:* cosine of 60 degrees is $1/2$.)

2. Consider the integral

$$\int_0^1 \int_0^{\sqrt{y}} 4x \cos(y^2) dx dy$$

(a) (5 points) Sketch the region of integration. (Please label everything.)

(b) (5 points) Use your sketch in (a) to reverse the order of integration.

(c) (10 points) Evaluate either the original integral or your answer in (b).

3. For constant b with $b \neq 1$ and $b \neq -1$, let

$$f(x, y) = (x - 1)^2 + (y - 2)^2 - 2bxy.$$

(a) (10 points) Find the x- and y- coordinates of the critical point. Your answer will be in terms of b.

(b) (5 points) Classify the critical point for all values of b with $b \neq 1$ and $b \neq -1$.

(c) (10 points) Sketch level curves of $f(x, y)$ if $b = 0$. Also, sketch the constraint $x^2 + y^2 = 1$. (Label everything.) Use your graph to mark where there appears to be a minimum of $f(x, y)$ subject to the constraint $x^2 + y^2 = 1$. Explain your reasoning.

(d) (10 points) Precisely determine the point in part (c) by using the method of Lagrange multipliers.

4. A circular lake 5 km in radius has a circular island 1 km in radius at its center. At r kilometers from the center of the island the depth of the lake is $d(r) = \frac{1}{10}(r-1)(5-r)$ kilometers, where $1 \le r \le 5$.

   (a) (5 points) Sketch the solid region in 3-space representing the water in the lake.

   (b) (10 points) Calculate the volume of water in the lake.

5. (a) (5 points) Suppose $f_x(2,1) = 3$, $f_y(2,1) = 10$, and $f(2,1) = 1$. Estimate the function $f(x,y)$ at the point (2.1,1.1).

(b) (5 points) The table below gives values of a function $f(x,y)$, which is increasing in x and decreasing in y on the region $R : 1 \leq x \leq 1.2, 2 \leq y \leq 2.4$. Find a Riemann sum which is a reasonable estimate for $\int_R f(x,y)dA$ with $\Delta x = 0.1$ and $\Delta y = 0.2$. Show your work.

		x		
		1.0	1.1	1.2
	2.0	5	7	10
y	2.2	4	6	7
	2.4	3	5	4

(c) (5 points) Suppose $\operatorname{div}\vec{F}(1,2,1) = 3$. Estimate the flux out of a small box of side 0.2 centered at the point $(1,2,1)$ and with edges parallel to the edges.

(d) (10 points) Discuss the accuracy of your estimations in parts (a)–(c). Are your estimates accurate? Is there anything you can assume to ensure that your estimates are accurate?

6. Let $\vec{F} = x\vec{i} - 2y\vec{j} + zy\vec{k}$ and $0 \le a, b, c \le 10$.

    (a) (10 points) Find the flux of $\vec{F}$ out of the rectangle solid $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

    (b) (5 points) Is $\vec{F}$ a curl vector field? Explain.

7. Consider the vector field

$$\vec{F}(x, y, z) = \frac{-z}{y^2 + z^2}\vec{j} + \frac{y}{y^2 + z^2}\vec{k}.$$

(a) (10 points) Compute the circulation, $\int_{C_1} \vec{F} \cdot d\vec{r}$, around the circle C_1 in the yz-plane of radius 1, centered at the origin, and oriented counterclockwise when viewed from the positive x-axis.

(b) (5 points) Calculate curl$\vec{F}$. Is $\vec{F}$ a gradient vector field? Explain. (*Be careful.*)

(c) (10 points) Calculate the circulation, $\int_{C_2} \vec{F} \cdot d\vec{r}$, around the ellipse C_2 given by $\frac{y^2}{9} + \frac{z^2}{16} = 1$, in the yz-plane, centered at the origin, and oriented counterclockwise when viewed from the positive x-axis. Explain your reasoning. (*Hint:* Use your answers from part (a) and (b).)