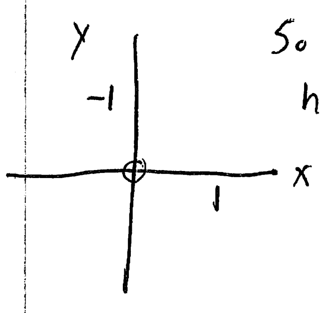
Solutions to Questions 1–4 Prelim 1
Math 2130 Spr '15

1. Along the x-axis (where $y=0$), $f(x,y) = \frac{x^2}{x^2} = 1$ except at the origin, where $f(x,y)$ is undefined.



So the limit as we approach the origin horizontally is $+1$.

Similarly, along the y-axis (where $x=0$),
$$f(x,y) = \frac{-y^2}{y^2} = -1$$
and the limit as we approach the origin vertically is -1.

Since limits, if they exist, must be unique, the limit as we approach the origin does not exist.
<u>Remark</u> $f(0,0)$ undefined does not mean the limit does not exist. It does mean $f(x,y)$ is not continuous there.

2. a) If $(4t-1, -2, 2t+3) = \lambda(2,-4,8)$, looking at the y-components tells us $\lambda = 1/2$.

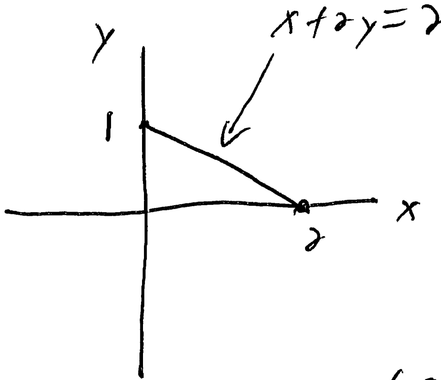
Then solving the equations $(4t-1) = \frac{1}{2}(2)$ and $2t+3 = \frac{1}{2}(8)$, we see that $t = 1/2$ solves both of them.
So the vectors are parallel exactly when $t = 1/2$

b) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -\hat{i}+\hat{j}-\hat{k}$ or $(-1,1,-1)$.

3. $f_x = -2x - 5y$
$f_y = -8y - 5x$

$$\begin{bmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ -5 & -8 \end{bmatrix}$$

Since the determinant D of this matrix is $(-2)(-8) - (-5)^2 < 0$, we have a saddle point at the origin.

4. 

$f(x,y) = xy$

First look for critical points:

$f_x = y = 0$ so $(0,0)$ is a
$f_y = x = 0$ critical point with $f(0,0)=0$.
(Since it is on the boundary, we'd have to further consider the point $(0,0)$ anyway.)

Second, look at the vertices:

$f(0,0) = f(0,1) = f(2,0) = 0$

Third, look at each of the three edges using 1-var calculus or LM
(s since we already did the vertices)

horizontal edge: $(x,0)$ $0<x<2$ $f(x,0)=0$ (all points are potential max/min)

vertical edge: $(0,y)$ $0<y<1$ $f(0,y)=0$ "

slant edge: $x = 2-2y$ $0<y<1$

$f(2-2y,y) = 2y-2y^2 = g(y)$

1-var calculus on $g(y)$ gives $g'(y) = 2-4y = 0$
$y = \frac{1}{2} \Rightarrow x = 2-2y = 2-1 = 1$

So $(1, \frac{1}{2})$ is also a possible max/min
$f(1,\frac{1}{2}) = \frac{1}{2}$

Comparing values we see that the global max is $\frac{1}{2}$ at $(1,\frac{1}{2})$ and the global min is 0 attained at all points on the two legs of the right triangle.