

Math 2130 Spring 2013 Prelim 3

Each question is worth 25 points. Good luck.

1. Consider the double integral

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dx dy.$$

a) (15 pts) Rewrite the double integral as an equivalent double integral in polar coordinates.

b) (10 pts) Evaluate the double integral.

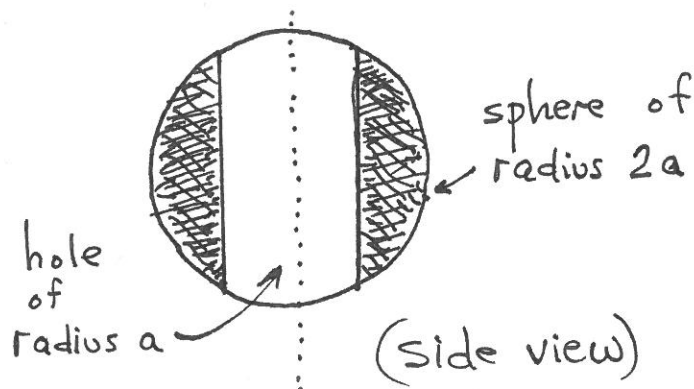
2. Consider the vector field $\vec{v} = x\vec{i} - y\vec{j}$.

a) (5 points) Sketch the vector field.

b) (5 pts) Sketch the flow line that passes through the point (1,1).

c) (15 pts) Find the parametric equations for the flow line $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ that passes through the point (1,1) at time $t = 0$.

3. A cylindrical hole of radius a is drilled symmetrically through a solid sphere of radius $2a$, as shown here.



Find the volume of the remaining solid, as follows:

a) (15 points) Express the volume of the remaining solid as a triple integral in cylindrical coordinates.

b) (10 points) Evaluate the triple integral.

4. Let C denote the plane curve $\vec{r}(t) = (e^t \cos t)\vec{i} + (e^t \sin t)\vec{j}$, for $0 \leq t \leq 2\pi$.

a) (10 pts) Sketch the curve C in the xy plane.

b) (15 points) Evaluate the line integral along C given by

$$\int_C \frac{x dx + y dy}{(x^2 + y^2)^{3/2}}.$$