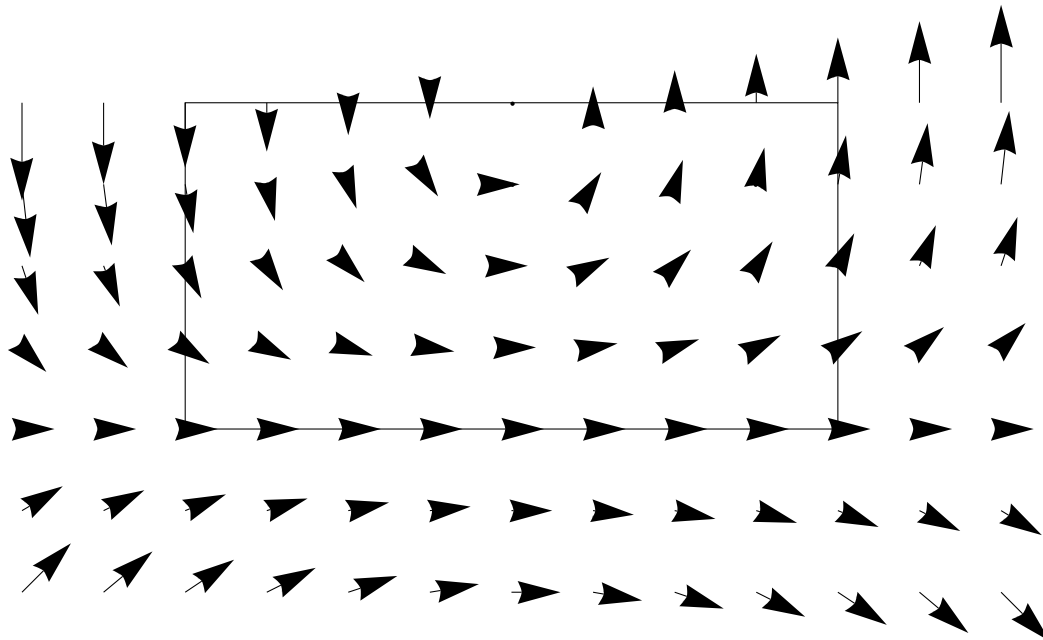


Math 2130 Homework 10: 18.1-18.3

- (1) Shown below is a rectangle (oriented counterclockwise) and a vector field \vec{F} . For each edge of the rectangle (bottom, right, top, left), is the vector line integral of \vec{F} over the edge positive, negative, or zero? Is the circulation of \vec{F} over the rectangle positive, negative, zero, or would you need to do a precise computation to tell? Can you conclude from this whether \vec{F} is path-independent?



- (2) The vector field shown above is given by the formula $\vec{F}(x, y) = (2 - y, xy)$. Set up the integral and evaluate the circulation of \vec{F} over the unit circle, oriented counterclockwise. Can you conclude from this whether \vec{F} is path-independent?
- (3) Consider the vector field $\vec{F}(x, y) = (1, x)$. Let R be the quarter circle $x \geq 0, y \geq 0, x^2 + y^2 \leq 1$. Set up the integral and evaluate the circulation of \vec{F} around the boundary of R , oriented counterclockwise.
- (4) Find some function $f(x, y)$ such that $\nabla f = (y^2 + 3, 2xy)$. Use the fundamental theorem of line integrals to compute $\int_C (y^2 + 3, 2xy) \cdot d\vec{r}$ where C is the curve parameterized by $\vec{r}(t) = (t, t^2)$ for $0 \leq t \leq 2$.
- (5) For a general vector field \vec{F} , how are the vector line integral of \vec{F} over the curve parameterized by $\vec{r}(t) = (\cos t, \sin t)$ for $0 \leq t \leq \pi$ and the vector line integral of \vec{F} over the curve parameterized by $\vec{r}(t) = (t, \sqrt{1 - t^2})$ for $-1 \leq t \leq 1$ related?