Math 2130 Homework 12: 19.3, 21.1, Scalar Surface Integrals

(1) Compute the divergence of the vector field $\vec{F}(x,y) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right)$.

(2) Compute the flux of the vector field $\vec{F}(x,y) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right)$ out of the unit circle.

(3) Use the divergence theorem to find the flux of the vector field $\vec{F}(x,y) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right)$ out of the square with vertices $(-2,-2), (-2,2), (2,2), (2,-2)$.

(4) Set up (but do not evaluate) the flux integral out of the bottom edge of the square above.

(5) Parameterize the plane passing through $(1,0,0), (0,1,0)$, and $(0,0,1)$. For what values of the parameters s,t is $\vec{r}(s,t)$ in the region $x \geq 0, y \geq 0, z \geq 0$?

(6) Parameterize the surface of a sphere of radius 2 centered at $(1,0,0)$.

(7) Parameterize the paraboloid $z = x^2 + y^2$ using the formula for a graph and the formula for a surface of revolution. What do the parameter curves of each of these parameterizations look like?

(8) Parameterize the donut given by revolving the circle of radius 1 centered at $(2,0)$ around the z-axis.

(9) Set up and evaluate an integral to find the average z coordinate of the conical surface $z = 2 - 2\sqrt{x^2+y^2}$ for $z \geq 0$.