Math 2130 Workshop: Calc 3 Prerequisites

Your workshop grades are based on participation: you will get a perfect score on this assignment if it appears you spent the class period on it. If you cannot solve one of the problems, you should review the corresponding material in the next week. Please come to office hours or visit the Math Support Center.

You are heavily encouraged to work with your fellow students on this assignment.

Please do not use a calculator on this particular assignment.

This workshop is intentionally long. Do not panic if you run out of time.

1) Compute $\frac{d}{dt}\sqrt{t^2+1}$.

You need to apply the chain rule: $\frac{1}{2\sqrt{t^2+1}} \cdot 2t$.

2) Show that $\sin^{-1}(x)$ and $\csc(x)$ are different functions.

To show that two functions are different, we need to plug in a value and see that they're different. $\sin^{-1}(0)$, or $\arcsin(0)$ is 0. $\csc(0)$, however, is undefined.

3) Find the absolute extremum of xe^x. Is it an absolute max or min?

Note that $\frac{d}{dx}xe^x = (x+1)e^x$. Since e^x is always positive, this is positive for $x > -1$ and negative for $x < -1$. This means that the function is decreasing on the interval $(-\infty, -1)$ and increasing on the interval $(-1, \infty)$. So at $x = -1$, xe^x takes on a global minimum of $-e^{-1}$.

4) Solve $x^2 - 2x - 4 = 0$.

Don't forget the quadratic equation! The solutions are $1 - \sqrt{5}$ and $1 + \sqrt{5}$.

5) Find both solutions to $x^x = x^2$.

First, we rule out the case where $x = 0$, since the left hand side is undefined for $x = 0$. Thus, the right hand side is positive, meaning we can take the natural log of both sides, giving us the equation: $\ln(x^x) = \ln(x^2)$, or $x\ln(x) = 2\ln(x)$, or $(x-2)\ln(x) = 0$. This has solutions when $x = 2$ or $\ln(x) = 0$ (i.e. $x = 1$).

6) An ellipse has two foci A and B, and is given by the set of all points P such that the sum of the distance from A to P plus the distance from P to B is constant. Locate the foci of the ellipse $x^2 + 2y^2 = 1$.

Recall that the foci of an ellipse are located symmetrically on the major axis, within the ellipse. That is, $A = (c, 0)$ and $B = (-c, 0)$ We know the ellipse contains the points $(1, 0)$ and$(0, \frac{1}{\sqrt{2}})$. As such, if we let r denote the common distance sum, we have the equation:

$$r = (1 - c) + (1 - -c) = \sqrt{c^2 + (\frac{1}{\sqrt{2}})^2} + \sqrt{c^2 + (\frac{1}{\sqrt{2}})^2}$$

Or:

$$2 = 2\sqrt{c^2 + \frac{1}{2}}$$

Or:

$$c = \pm\frac{1}{\sqrt{2}}$$

7) Compute $\int_0^1 xe^x\, dx$.

Let us first compute $\int xe^x\, dx$ by parts:

$$\int xe^x\, dx = xe^x - \int e^x\, dx = xe^x - e^x + C$$

Now, evaluate between the bounds:

$$\int_0^1 xe^x\, dx = [xe^x - e^x]_0^1 = (e - e) - (0 - 1) = 1$$

8) Compute $\int \frac{x}{x^2+1}\, dx$.

This is a u-substitution problem. If we let $u = x^2 + 1$, then $du = 2x\, dx$, and our integrand becomes:

$$\int \frac{x}{x^2 + 1}\, dx = \int \frac{1}{u}\frac{du}{2} = \frac{\ln|u|}{2} + C = \frac{1}{2}\ln|x^2 + 1| + C$$

(Note that technically, any function of the form $f(x) = \begin{cases} \ln(-x) + A & x < 0 \\ \ln(x) + B & x > 0 \end{cases}$ is an antiderivative for $\frac{1}{x}$. Check it for yourself.)

9) Find the point on the curve $y = \ln x$ such that the tangent line at that point passes through the origin.

The slope of the tangent line is given by $m = \frac{1}{x}$. Suppose we wind up drawing the tangent line at the point $x = a$. Then the slope of the tangent line is $\frac{1}{a}$ and the tangent line passes through the point $(a, \ln(a))$. As such, the equation of the tangent line is $y = \frac{1}{a}(x - a) + \ln(a)$. If we want this line to pass through the origin, y must be 0 when $x = 0$. Plugging in, we get $0 = \frac{-a}{a} + \ln a$ which happens when $\ln(a) = 1$, or $a = e$.

10) Compute the upper Riemann Sum of $\sin(x)$ over the interval $[0, \pi]$ using three equal width intervals.

The width of our rectangles should be $\frac{\pi}{3}$ and their heights are $\sin(\frac{\pi}{3}), \sin(\frac{\pi}{2})$ and $\sin(\frac{2\pi}{3})$ which are the maximum values on each interval. These are, $\frac{\sqrt{3}}{2}, 1$, and $\frac{\sqrt{3}}{2}$ respectively. Our Riemann sum is the sum of the areas of these rectangles, $\frac{\pi}{3}(1 + \sqrt{3})$.