HW 10 SOLUTIONS MATH 2210

RAKVI

1. Problem 6.4.22

We may assume that $\{u_1, \ldots, u_p\}$ is an orthomormal basis for W. Let U be the matrix whose columns are $u_1, \ldots, u_p$. Then,

$$T(x) = (UU^T)(x).$$

Since, T is a matrix transformation it is a linear transformation.

2. Problem 6.4.23

Given $A = QR$, partition A as $[A_1\ A_2]$ where A_1 has p columns. Partition Q as $[Q_1\ Q_2]$ where Q_1 has p columns, and partition R as

$$\begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix},$$

where R_{11} is a $p \times p$ matrix. Then,

$$A = [A_1 A_2] = [Q_1 R_{11} Q_1 R_{12} + Q_2 R_{22}].$$

Observe that $Q_1 R_{11}$ is a QR factorization of A_1.

3. Problem 6.5.8

From exercise 4,

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix},$$

$$b = \{5, 1, 0\},$$

and

$$\hat{x} = [1, 1].$$

Since, $A\hat{x} - b = [-1, -1, 2]$ the least squares error is $\sqrt{6}$.

4. Problem 6.5.11

a. Because the columns of A are orthogonal, the method of example 4 may be used to find

$$\hat{b} = [3, 1, 4, -1].$$

b. The vector $\hat{x}$ contains the weights which must be placed on a_1, a_2, a_3 to produce $\hat{b}$. So,

$$\hat{x} = [2/3, 0, 1/3]$$

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5. Problem 6.5.16

The least squares solution satisfies $R\hat{x} = Q^T b$. Solving this system of equation gives us

$$\hat{x} = [2.9, 0.9]$$

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6. Problem 6.5.18

a. True
b. False. See figure 1 and the paragraph preceding it.
c. True.
d. False. See theorem 14.
e. False. See comments after example 4.
f. False.

7. Problem 6.5.22

By the rank theorem,

$$rankA^T A = n - dimNulA^T A = n - dimNulA = rankA$$

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8. Problem 6.5.24

Since in this case $A^T A = I$, the normal equations give

$$\hat{x} = A^T b$$

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