Math 2210 Homework 1 Solutions

1.1

10. $(-3, -5, 6, -3)$

16. Consistent

22. $h = -5/3$

24. a. True. See the box preceding the Subsection titled "Existence and Uniqueness Questions"
    b. False. The definition of row equivalent requires that there exists a sequence of row operations that transforms one matrix into the other.
    c. False. By definition, an inconsistent system has no solution.
    d. True. This definition of equivalent systems is in the second paragraph after the equation (2).

26. There are many possible answers. For example the systems corresponding matrices each have the solution set $x_1 = -2$, $x_2 = 1$, $x_3 = 0$ (where tildes represent row equivalences).

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 2 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 2 & 1 & 0 & -3 \\ 2 & 0 & 1 & -4 \end{bmatrix}$$

28. $d - c(b/a) \neq 0$, or $ad - bc \neq 0$.

33.
$$\begin{array}{lllll} 4T_1- & T_2 & & -T_4 & = 30 \\ -T_1+ & 4T_2 & -T_3 & & = 60 \\ & -T_2+ & 4T_3- & T_4 & = 70 \\ -T_1 & & -T_3+ & 4T_4 & = 40 \end{array}$$

1.2

6.
$$\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} \blacksquare & * \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

12. $\begin{cases} x_1 = 5 + 7x_2 - 6x_4 \\ x_2 \text{ is free} \\ x_3 = -3 + 2x_4 \\ x_4 \text{ is free} \end{cases}$

16. a. A unique solution

    b. Consistent, with many solutions

18. $h \neq -15$

23. Yes. The system is consistent because with three pivot, there must be a pivot in the third (bottom) row of the coefficient matrix. The reduced echeelon form cannot contain a row of the form [0 0 0 0 0 1].

26. Since there are three pivots (one in each row) the augmented matrix must reduce to the form

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix} \text{ and so } \begin{array}{rcl} x_1 & & = a \\ & x_2 & = b \\ & & x_3 = c \end{array}$$

For any values of a, b, and c, the solution exist and is unique.

30. Example:

$$x_1 + x_2 + x_3 = 4$$

$$2x_1 + 2x_2 + 2x_3 = 5$$