HW 6 SOLUTIONS MATH 2210

RAKVI

1. Problem 3.1.8

Expanding along the first row we get, determinant = 4(6)-1(11)+2(-8)=-3.

2. Problem 3.1.14

First expand along either the fourth row or fifth column. Then expand along the third row and finally expand along the first column or second row to get that determinant = (2)(3(-11)+2(18))=6.

3. Problem 3.2.8

$$\begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 1 & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -15 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

Last operation is row interchange so determinant of original matrix is -10.

4. Problem 3.2.24

The vectors are linearly dependent, because determinant of the matrix (whose columns are the vectors in question) is 0.

5. Problem 3.2.28

a. False, see theorem 3.
b. False. See the paragraphs following example 2.
c. False. See example 3.
d. False. See theorem 5.

6. Problem 3.2.32

$det(A^3) = det(A)^3$, since $det(A^3) = 0$, $det(A) = 0$ and therefore, A is not invertible.

7. 3.2.36

Key idea is that if you multiply a row by r then determinant gets multiplied by r. Therefore, $det(rA) = r^n det(A)$.

8. Problem 3.2.40

a. $det(AB) = det(A)det(B) = 3$
b. $det(B^5) = det(B)^5 = -1$
c. $det(2A) = 16det(A) = -48$
d. $det(A^TBA) = det(A)^2det(B) = -9$
e. $det(B^{-1}AB) = det(A) = -3$

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9. Problem 3.3.10

We can compute $det(A_1 b) = 4s + 4$ and $det(A_2 b) = -2s$. Observe that $det(A) = 4s(s+2)$ is equal to 0 for $s = 0, -2$. So, the system will have unique solution for s not equal to 0 or -2. In that case the solution will be $x_1 = 4s + 4/4s(s+2) = s + 1/s(s+2)$, $x_2 = -2s/4s(s+2) = -1/2(s+2)$.

10. Problem 3.3.18

Each cofactor of A is an integer. Since, det(A)=1, A^{-1} will have integer entries.

11. Problem 3.3.22

Subtract (0,-2) from each vertex to get new parallelogram with vertices (0,0),(5,0),(-3,3) and (2,3). Area of this parallelogram is same as original parallelogram and is determined by determinant of matrix with columns (5,0), (-3,3) which is 15.

12. Problem 3.3.28

Area of S is 4. Determinant of matrix A which represents T is 3. So, area of T(S) is 12.

13. Problem 3.3.30

Translate the vertices so that one of the vertex is origin and use the formula for area of triangle. Then, use row reduction to conclude that both sides are equal.

14. Problem 5.1.6

Compute Ax to check whether Ax= cx for some constant c. If yes, then c is the eigenvalue and x is the eigenvector corresponding to that eigenvalue.

15. Problem 5.1.16

For $\lambda = 4$, just compute the basis for nullspace of $A - \lambda I$ which is $\{[2, 3, 1, 0], [0, 0, 0, 1]\}$.

16. Problem 5.1.22

a. False. Eigenvector must be non-zero.
b. False. Refer to example 4.
c. True. See the paragraph after example 1.
d. False.
e. True. See paragraph after example 3.

17. Problem 5.1.26

Let λ be an eigenvalue of A. So $Ax = \lambda x$ for some nonzero x, multiply this equation by A to get $A^2 x = (\lambda)^2 x$, now A^2 is 0 matrix so, λ must be 0.

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