HW 7 SOLUTIONS MATH 2210

RAKVI

1. Problem 5.2.12

We want to compute $det(A - \lambda I)$. Make a cofactor expansion along the third row to get

$$det(A - \lambda I) = -\lambda^3 + 5\lambda^2 - 2\lambda - 8$$

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2. Problem 5.2.20

$$det(A^T - \lambda I) = det(A^T - \lambda I^T) = det((A - \lambda I)^T) = det(A - \lambda I)$$

.

3. Problem 5.2.22

a. False, see the paragraph before theorem 3.
b. False. See theorem 3.
c. True. See the paragraph before example 4.
d. False. See the warning after theorem 4.

4. Problem 5.3.16

The eigenvalues of A are 2 and 1. For $\lambda = 2$, two linearly independent eigenvectors are $\{[-2, 1, 0], [-3, 0, 1]\}$. For $\lambda = 1$, a eigenvector is $\{[-2, -1, 1]\}$. From these eigenvectors we construct

$$P = \begin{bmatrix} -2 & -3 & -2 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

and

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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5. Problem 5.3.22

a. False, see the diagonalization theorem.
b. False. See example 3.
c. True.
d. False. See example 4.

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6. 5.3.28

If A has n linearly independent eigenvectors, then $A = PDP^{-1}$ for some invertible matrix P and diagonal matrix D. Since, $A = PDP^{-1}$, $A^T = QDQ^{-1}$ where $Q = (P^T)^{-1}$. Hence, A^T is diagonalizable and columns of Q are n linearly independent eigevectors of A^T.

7. Problem 5.3.32

Consider

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

. Its a nondiagonal matrix and it is diagonalizable because it has two distinct eigenvalues 0 and 1. It is not invertible because determinant is 0.

8. Problem 5.4.10

a. Use the fact that **(p+q)**(a)=**p**(a)+**q**(a) and **(cp)**(a)=c.**p**(a) to show that T is a linear transformation.
b. Compute $T(1), T(t), T(t^2), T(t^3)$ to get a matrix of transformation as

$$\begin{bmatrix} 1 & -3 & 9 & -27 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \end{bmatrix}$$

9. Problem 5.4.22

If A is diagonalizable then $A = P * D * P^{-1}$ for some P. If B is similar to A then $B = Q * A * Q^{-1}$ for some Q. Therefore, $B = QP * D * QP^{-1}$. Hence, B is diagonalizable.

10. Problem 5.4.27

For each j, $I(b_j) = b_j$. Since the standard coordinate vector of any vector in $\mathbb{R}^n$ is the vecto itself $[I(b_j)]_E = b_j$. Thus, the required matrix is $[b_1\ b_2 \dots b_n]$.

11. Problem 5.5.4

Eigenvalues of A are $4 \pm \iota$. For $4 + \iota$ basis for eigenspace is $[1 + \iota, 1]$. For $4 - \iota$ basis for eigenspace is $[1 - \iota, 1]$.

12. Problem 5.5.8

Eigenvalues of A are $\sqrt{3} \pm 3\iota$. Scale factor of transformation (by A) is $2\sqrt{3}$ (it is the absolute value of eigenvalue). The angle of rotation is $\arctan(-3/\sqrt{3})$=-$\pi/3$ radians.

13. Problem 5.5.16

Eigenvalues of A are $4 \pm \iota$. By theorem 9,

$$P = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

and

$$C = P^{-1}AP = \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}$$

14. Problem 5.5.24

Since x is an eigenvector $\bar{x}^T Ax = \lambda\bar{x}^T x$. It is easy to see that $\bar{x}^T x$ is real and positive. Since, $\bar{x}^T Ax$ is real, so is λ. Write $x = u + \iota v$. Then, the real part of Ax is Au and real part of λx isλu. Therefore, $Au = \lambda u$.

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