SOLUTIONS TO HOMEWORK 2

Section 1.3 Vector Equations.

18. $h = \frac{-7}{2}$

24.

a: True. See the beginning of subsection "Vectors in $\mathbb{R}^n$."
b: True. Use Figure 7 to draw the parallelogram determined by $\mathbf{u} - \mathbf{v}$ and $\mathbf{v}$.
c: False. See the first paragraph of the subsection "Linear Combinations."
d: True. See the statement that refers to Figure 11.
e: True. See the paragraph following the definition of Span $\{\mathbf{v_1}, \cdots, \mathbf{v_p}\}$.

30. Let m be the total mass of the system. By definition,

$$\mathbf{v} = \frac{1}{m}(m_1\mathbf{v_1} + \cdots + m_k\mathbf{v_k}) = \frac{m_1}{m}\mathbf{v_1} + \cdots + \frac{m_k}{m}\mathbf{v_k}.$$

The second expression displays $\mathbf{v}$ as a linear combination of $\mathbf{v_1}, \cdots, \mathbf{v_k}$, which shows that $\mathbf{v}$ is in Span$\{\mathbf{v_1}, \cdots, \mathbf{v_k}\}$.

Section 1.4: The matrix equation $A\mathbf{x} = \mathbf{b}$.

4. $$A\mathbf{x} = \begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1 \cdot \begin{bmatrix} 8 \\ 5 \end{bmatrix} + 1 \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} + 1 \cdot \begin{bmatrix} -4 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

$$A\mathbf{x} = \begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \cdot 1 + 3 \cdot 1 + (-4) \cdot 1 \\ 5 \cdot 1 + 1 \cdot 1 + 2 \cdot 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

14. No. The equation $A\mathbf{x} = \mathbf{u}$ has no solutions.

32. A set of three vectors cannot span $\mathbb{R}^4$. Reason: The matrix A whose columns are these three vectors has four rows. To have a pivot in each row, A would have to have at least four columns (one for each pivot), which is not the case. Since A does not have a pivot in each row, its columns do not span $\mathbb{R}^4$, by Theorem 4. In general, a set of n vectors cannot span $\mathbb{R}^m$ when n is less than m.

34. If the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution, then the associated system of equations does not have any free variables. If every variable is a basic variable, then each column of A is a pivot column. So the reduced echelon form of A must be $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Now it is clear that A has a pivot position in each row. By Theorem 4, the columns of A span $\mathbb{R}^3$.

Section 1.5: Solution sets of Linear systems.

6. $$\mathbf{x} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

12. $$\mathbf{x} = x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ 0 \\ 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ -4 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

14. $\mathbf{x} = \begin{bmatrix} 0 \\ 8 \\ 2 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 1 \\ -5 \\ 1 \end{bmatrix} = \mathbf{p} + x_4\mathbf{q}$. The solution set is the line through $\mathbf{p}$ parallel to $\mathbf{q}$.

24.

- **a:** False. A nontrivial solution of $A\mathbf{x} = \mathbf{0}$ is any nonzero $\mathbf{x}$ that satisfies the equation. See the sentence before Example 2.
- **b:** True. See Example 2 and the paragraph following it.
- **c:** True. If the zero vector is a solution, then $\mathbf{b} = A\mathbf{x} = A\mathbf{0} = \mathbf{0}$.
- **d:** True. See the paragraph following Example 3.
- **e:** False. The statement is true only when the solution set of $A\mathbf{x} = \mathbf{b}$ is nonempty. Theorem 6 only applies to a consistent system.

30.

- **a:** Yes
- **b:** No