

Math 3040, Equivalence relations

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Let X be the set $\{(x, y) \mid x, y \in \mathbb{Z}\}$, where $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ is the set of integers. For each relation \sim on X , prove that it is an equivalence relation or prove which of the three properties of an equivalence relation don't hold.

1. $(x_1, y_1) \sim (x_2, y_2)$ if and only if $x_1 y_2 = x_2 y_1$.
2. $(x_1, y_1) \sim (x_2, y_2)$ if and only if $x_1 - y_1 = x_2 - y_2$.
3. $(x_1, y_1) \sim (x_2, y_2)$ if and only if $x_1 - y_1 = 2(x_2 - y_2)$.
4. $(x_1, y_1) \sim (x_2, y_2)$ if and only if $x_1 - y_1 = y_2 - x_2$.