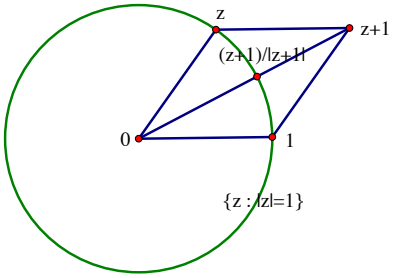
Math 3040, Take Home Prelim 2
Due April 29, 2014

Please hand in your solutions as a TeX document in class on Tuesday April 29, 2014, and be sure to cite the sources that you use in your answers.

1. In your own words, state the precise definition of what the limit of a real valued function of a real variable is, and give a rigorous proof that that the limit of the sum of two functions is the sum of the limits. That is, for functions f, g, assuming that the two righthand limits exist, prove
$$\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x).$$

2. (a) Suppose that $z \neq -1$ is a complex number of norm 1. That is $|z| = 1$. Prove that
$$\left(\frac{1+z}{|1+z|}\right)^2 = z.$$
Here it might be helpful to consider the following figure to provide a geometric proof.



(b) Suppose that $z = n + mi$ is a complex number, where n and m are integers, not both zero. Prove that $(z/|z|)^2$ has rational coordinates, lies on the unit circle, and any such point on the unit circle with rational coordinates is of this form. Hint: Use Part (a).

3. A conic in the plane is any curve of the form $ax^2 + by^2 + cxy + dx + ey + f = 0$, where a, b, c, d, e, f are real constants such that not all of a, b, c are zero. Prove that if 5 points are given in the plane such that no three are collinear, then there is a unique conic containing those points.

4. Suppose two triangles, lying on three parallel vertical lines, are given as in the Figure, where the red pair of corresponding sides are parallel. Then use Desargues' Theorem to prove that the line through the intersection of corresponding sides are parallel to the horizontal sides. In other words, prove that all three red lines in the Figure are parallel.