Math 3040, Some Probability Problems
Due March 20, 2014

The following problem is from "Pillow Problems and a Tangled Tale" by Lewis Carroll.

Problem: A bag contains 2 counters, as to which nothing is known except that each is either black or white. Ascertain their colours without taking them out of the bag.

Lewis Carroll's solution: One is black, and the other is white.

Lewis Carroll's explanation: We know that, if a bag contained 3 counters, 2 being black and on white, the chance of drawing a black one would be $\frac{2}{3}$; and that any *other* state of things would *not* give this chance.

Now the chances, that the given bag contains (α) $BB, (\beta)$ $BW, (\gamma)$ WW, are respectively $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

Add a black counter.

Then the chances, that it contains (α) $BBB, (\beta)$ $BWB, (\gamma)$ WWB, are as before, $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

Hence the chance, of now drawing a black one,

$$= \frac{1}{4} \cdot 1 + \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{4} \cdot \frac{1}{3} = \frac{2}{3}.$$

Hence the bag now contains BBW (since any *other* state of things would *not* give this chance).

Hence, before the black counter was added, it contained BW, i. e. one black counter and one white.

Q.E.F.

Can you explain this explanation?

A Game called "Say Red": This is betting game, where you have to invest $\$1 + \epsilon$ to play. I take a well-shuffled deck of cards an start turning them over one at a time. At some point you have to stop me an declare that the next card is red. You can do this any time during the game, declaring the first card or the last card or any card in between is red. You *have to say red*, and you can't quit without saying red some time, but you can make your decision depending on the cards you have seen. If you are correct and the card you choose is red, you win \$2. If the card is black, you loose your dollar $\$1 + \epsilon$.

Problem: How large an $\epsilon > 0$ are you willing to invest in this game?