Math 3040, Sample Proof
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Following is the infamous Cauchy-Schwarz inequality with the proof we discussed in class. A lot of the preamble in the TeX here can be simplified. There is a lot of help and templates on the web. Enjoy.

Here it is understood that you know the basic properties of a vector space and the properties of the dot product, i.e. the inner product.

Theorem 1. *In a real inner product vector space the following holds for all vectors* $\mathbf{v}$, $\mathbf{w}$

$$\mathbf{v} \cdot \mathbf{w} \leq |\mathbf{v}||\mathbf{w}|. \tag{1}$$

Proof. There are two cases. Case 1: Either $\mathbf{v} = 0$ or $\mathbf{w} = 0$ in which case both sides of (1) are 0.

Case 2: Both $\mathbf{v} \neq 0$ and $\mathbf{w} \neq 0$. So $|\mathbf{v}| \neq 0$ and $|\mathbf{w}| \neq 0$. Thus

$$0 \leq \left(\frac{\mathbf{v}}{|\mathbf{v}|} - \frac{\mathbf{w}}{|\mathbf{w}|}\right) \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|} - \frac{\mathbf{w}}{|\mathbf{w}|}\right) = \frac{\mathbf{v} \cdot \mathbf{v}}{|\mathbf{v}|^2} + \frac{\mathbf{w} \cdot \mathbf{w}}{|\mathbf{w}|^2} - 2\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}||\mathbf{w}|} = 2 - 2\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}||\mathbf{w}|}.$$

Clearing fractions and moving the negative term to the left we get (1). $\square$