

MATH 3040
Assignment 5

1. Let A be the left set; let B be the right set. We have to show that $A \subseteq B$ and $B \subseteq A$.
To show that $A \subseteq B$, let $x \in A$. Then $x \leq a$ and $\min(x, a) \leq b$. Either $a \leq b$ or $b \leq a$. If $a \leq b$ then $x \leq a = \min(a, b)$, thus $x \in B$. For $b < a$, we know that $x \leq b$ since $x \leq a$ and $x = \min(x, a) \leq b$. Then $x \leq b = \min(a, b)$. Therefore $x \in B$.
To show $B \subseteq A$, let $x \in B$. Then $x \leq \min(a, b)$. Either $a \leq b$ or $b < a$. Similar to the process above, you should show by yourself here that $x \in A$. Therefore $A = B$.
2. (a) When $x = \pm 1, 0$, P_x is a finite set
(b) $\cap_{0 < x < 1} P_x = \emptyset$, $\cup_{0 < x < 1} P_x = (0, 1)$
(c) $\cap_{k=1}^N P_{2^k} = \{2^{a \cdot \text{lcm}(1, 2, \dots, N)} \mid a \in \mathbb{N}\}$, $\cap_{k=1}^{\infty} P_{2^k} = \emptyset$
3. (a) The largest set on which f is defined in $\{x \in \mathbb{R} \mid x \neq \frac{-d}{c}\}$.
(b) Let $x, x' \in \mathbb{R}$ such that $f(x) = f(x')$. Then $\frac{ax+b}{cx+d} = \frac{ax'+b}{cx'+d}$, which can be simplified as $(x-x')(ad-bc) = 0$. If $ad \neq bc$, then $x = x'$.
(c) The inverse function is $y = \frac{b-dx}{cx-a}$ for $x \neq \frac{a}{c}$. Hence the range of f is $y \in \mathbb{R} \mid y \neq \frac{a}{c}$.
4. For $f(x) = \frac{x}{1-x^2}$, $g(y) = \frac{-1 + \sqrt{1+4y^2}}{2y}$, by Theorem 3.4, you have to show $g(f(x)) = x$ and $f(g(x)) = x$.
5. $f^{-1}(\{k \in \mathbb{Z} \mid k \leq 0\}) = \{0, 1, 2\}$. Make sure among the numbers you found from $f^{-1}(\{-7, -6, -5, -4, -3, -2, -1, 0\})$ only the integers qualify.