MATH 3040
Assignment 6

**1.** $\aleph_1 < \aleph_2$ by Theorem 4. Still by Theorem 4, $\#\mathbb{N} < \#\mathcal{P}(\mathbb{N})$. And since $\#\mathbb{R} = \#\mathcal{R}(\mathbb{N})$, we have $\aleph_0 < \aleph_1$. Therefore $\aleph_0 < \aleph_1 < \aleph_2$.

**2.** $X_0 = \{2n + 1 \mid n \in \mathbb{N}\}$

**3.** $C = (-\infty, 1]$

**4.** The cardinality of $\{(n_1, n_2, \ldots, n_k, \ldots) \mid n_k \in \mathbb{N}\}$ is $\aleph_1$. Try to set up a correspondence, that is, a bijection between $\{(n_1, n_2, \ldots, n_k, \ldots) \mid n_k \in \mathbb{N}\}$ and $\mathcal{P}(\mathbb{N})$.

**5.** The cardinality of $\{(n_1, n_2, \ldots, n_k) \mid n_k \in \mathbb{N}\}$ is $\aleph_0$. We know $\#\mathbb{N} = \#\mathbb{N}^2$. And hence we have $\#\mathbb{N} = \#\mathbb{N}^k$. Now think about the reason why this equation leads to the answer of the problem.