Math 3040, Prelim I Practice Problems
February 25, 2014

Outline of some of the subjects to be covered on the Prelim:

- Mathematical Induction
- Recursively defined sequences
- Rational and irrational numbers
- Real inequalities
- Vector spaces and vector operations
- Equivalence relations

Here are some problems to discuss on Tuesday:

1. Use mathematical induction to show that the number of ways of choosing two elements from a set of n distinct elements is $n(n-1)/2$.
2. For x a positive real number, prove that $1 < x$ if and only if $0 < x^2 - x - 2$.
3. Negate the condition "The real number x is such that $x^2 - 2 > 0$ and $x \leq \sqrt{2}$".
4. (a) Define the Fibonacci sequence F_n.
   (b) Prove that for all $a > 0$ and all but a finite number of n, $n^a < F_n$.
5. There are two irrational numbers a, b such that ab is rational and $a + b$ is rational. Prove that for any such pair a, b, they are the solution to a quadratic polynomial with integer coefficients.
6. Use the Cauchy-Schwarz inequality to show that the mean of n real numbers is less than or equal to the root-mean-square of those numbers. In other words, show that

$$\frac{1}{n}\sum_{i=1}^{n} x_i \leq \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}.$$

7. Define $a \sim b$ for two real numbers if $a - b$ is rational. Prove that $\sim$ is and equivalence relation.