Main part

1. (10) (A 11) Find all the singular points of the system \( \dot{x} = xy + 12, \ \dot{y} = x^2 + y^2 - 25 \). Study their stability, determine the types of singular points, and sketch the phase curves near the singular points.

2. (10) (A 12) Find all the singular points of the system \( \dot{x} = -\sin y, \ \dot{y} = \sin x + \sin y \). Study their stability, determine the types of singular points, and sketch the phase curves near the singular points.

3. (8) Plot the orbits of a system written in polar coordinates: \( \dot{r} = r - r^2, \ \dot{\phi} = 1 \). Distinguish a limit cycle and find its multiplier.

4. (7) Find the approximations by the Euler broken lines with the step \( h = \frac{1}{k} \) to the solutions of the Cauchy problem \( \dot{x} = Ax, \ x(0) = x_0 \).

5. (15) Consider a planar vector field in the plane with a negative divergence. Prove that this field cannot have two closed orbits one of which lies in the domain bounded by another one.

Supplementary part

6. (15) Find the first derivative in \( \varepsilon \) at \( \varepsilon = 0 \) of a periodic solution, close to the identical zero, of an equation \( \dot{x} = -x + x^3 + \varepsilon \sin t \).

7. (10) Consider a linear center \( \dot{x} = Ax, x \in \mathbb{R}^2 \). Find a polynomial vector field \( f(x) = o(x) \) such that the singular point 0 of the equation \( \dot{x} = Ax + f(x) \) will be a) Lyapunov stable b) unstable.