Solve

1. (5) $x^{(3)} - 6\ddot{x} + 12\dot{x} - 8x = 0$

2. (5) $\dddot{x} + 2\ddot{x} + x = e^t$

Find a periodic solution with the frequency $\omega$, when exists, and plot the graph of its amplitude as a function of $\omega$:

3. (10) $\ddot{x} + x = \sin \omega t$

4. (10) $\dddot{x} - 0.1\ddot{x} + x = \sin \omega t$

Find a partial solution of the following equation:

5. (10) $x^{(4)} + 4x = e^t \sin \omega t$

6. (10) $x^{(3)} - 8x = e^t \cos \omega t$

Supplementary part.

7. (10) Prove that quasipolynomials with different exponents or different degrees are linear independent. Hint: use induction in the sum of degrees, and the division-derivation method.

8. (10) Find all the eigenfunctions and eigenvalues of the operator of taking the first derivative on the line.

9. (10) Find all the eigenfunctions and eigenvalues of the operator of taking the second derivative on the circle $S^1 = \mathbb{R}/2\pi\mathbb{Z}$. 