Math 4410
Fall 2009
Exam 2

Name:

Directions:

Complete all six questions.

Show your work. A correct answer without any scratch work or justification may not receive much credit.

You may not use any notes, calculators, or other electronic devices.

You have 75 minutes.

Problem 1: _____ / 10

Problem 2: _____ / 10

Problem 3: _____ / 10

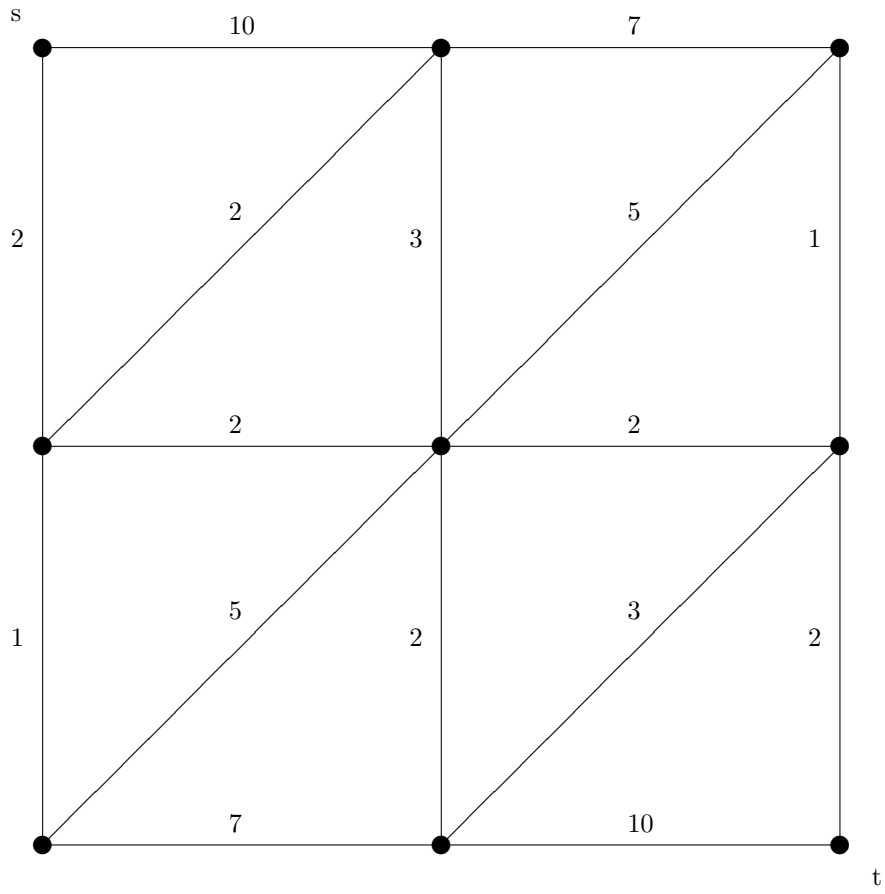
Problem 4: _____ / 10

Problem 5: _____ / 10

Problem 6: _____ / 10

Total: _____ / 60

1. Find a maximum flow and a minimum cut for the following transportation network. Interpret each edge shown as being an edge in each direction with the listed capacity.



2. Give a symmetric chain decomposition of B_4.

3. Give a De Bruijn sequence that gives all possible four digit sequences of 0's and 1's.

4. Give a matrix such that all entries are integers, each entry differs from the following matrix by less than 1, the sum of the entries in each column differs from the following matrix by less than 1, and the sum of the entries in each row differs from the following matrix by less than 1.

$$\begin{bmatrix} 4.2 & 3.6 & -1.8 \\ 5.3 & 0.3 & 2.4 \\ -1.5 & -3.9 & 6.4 \end{bmatrix}$$

5. Say that a bipartite graph with vertices $X = \{x_1, x_2, \ldots, x_m\}$ in one part and $Y = \{y_1, y_2, \ldots, y_n\}$ in the other part is *shifted* if, for every $a \leq b$ and $c \leq d$, if $\{x_b, y_d\}$ is an edge, then $\{x_a, y_c\}$ is also an edge. Show that if a shifted bipartite graph has a perfect matching of X into Y, then $\{x_m, y_1\}, \{x_{m-1}, y_2\}, \ldots, \{x_1, y_m\}$ is such a perfect matching.

6. Let P be a poset whose elements are subsets of $[9]$ of size three, with order defined such that $a \leq b$ if the smallest element of a is less than or equal to the smallest element of b, the second smallest element of a is less than or equal to the second smallest of b, and the largest element of a is less than or equal to the largest element of b. For example, $\{2, 6, 3\} \leq \{9, 5, 3\}$ because $2 \leq 3$, $3 \leq 5$, and $6 \leq 9$. As another example, $\{1, 3, 7\}$ is not comparable to $\{2, 4, 6\}$ because $3 < 4$ but $7 > 6$. If we wish to partition P into as few antichains as possible, determine the minimum number of antichains needed.