

Math 4410
Fall 2009
Exam 3

Name:

Directions:

Complete all six questions.

Show your work. A correct answer without any scratch work or justification may not receive much credit.

You may not use any notes, calculators, or other electronic devices.

You have 75 minutes.

Problem 1: _____ / 10

Problem 2: _____ / 10

Problem 3: _____ / 10

Problem 4: _____ / 10

Problem 5: _____ / 10

Problem 6: _____ / 10

Total: _____ / 60

1. Give an ordinary generating function for the sequence a_n defined by $a_0 = 1$, $a_1 = 3$, and $a_n = a_{n-1} + 2a_{n-2}$ for all $n \geq 2$.

2. For any positive integer n , show that there is some value of c (which can depend on n) such that for all $k \geq c$, $p_k(n+k) = p_c(n+c)$. Also find the minimum such value of c .

3. How many positive integers are there that are factors of at least one of 2^{437} , 3^{559} , and 2^{855} ?

4. Give a formula for the Stirling number of the second kind $S(n, 2)$.

5. Give an exponential generating function for the sequence a_n defined by $a_0 = a_1 = 1$ and $a_{n+1} = a_n + n(n-1)a_{n-1}$ for all $n \geq 1$.

6. Let P be the set of partitions of 150 such that for all $k \geq 1$, if there is a part of size $k + 1$, then there is at least one part of size k . Show that the number of partitions in P for which the largest part is even is equal to the number of partitions in P for which the largest part is odd.