

Math 4500 Warmup #11, due 2/22/2017

Name:

Student Number:

Read the Lecture Notes and Slides on the Exponential map.

Remember the 1/2 hour rule.

Exercise I. Compute the coefficients of t^0 , t^1 , t^2 , t^3 , of

$$e^{tX}e^{tY}.$$

Similarly for

$$e^{tX}e^{tY}e^{-tX}e^{-tY},$$

where X, Y are $n \times n$ matrices with complex entries.

Exercise II. List all 2-dimensional Lie algebra up to equivalence. For 3 dimensions, this means find all possible coefficients c_{ij}^k so that $[e_i, e_i] = 0$, and

$$\begin{aligned}[e_1, e_2] &= c_{12}^1 e_1 + c_{12}^2 e_2 + c_{12}^3 e_3 \\ [e_2, e_3] &= c_{23}^1 e_1 + c_{23}^2 e_2 + c_{23}^3 e_3 \\ [e_3, e_1] &= c_{31}^1 e_1 + c_{31}^2 e_2 + c_{31}^3 e_3.\end{aligned}$$

form a Lie algebra. Two Lie algebras are equivalent if they *differ* by a change of basis only.