Math 4500 Warmup #14, due 3/1/2017

Name: Student Number:

Review the notion of differential of a map and its relation to directional derivatives.

Remember the 1/2 hour rule.

Exercise I. Let $\mathfrak{g} \subset M(n,\mathbb{R})$ be a Lie subalgebra. Let V be a vector space complement, *i.e.* $M(n,\mathbb{R}) = \mathfrak{g} + V$ so that $V \cap \mathfrak{g} = (0)$. Define a map $\Phi : M(n,\mathbb{R}) = \mathfrak{g} + V \longrightarrow M(n,\mathbb{R})$ be given by $\Phi(X,Y) = e^X \cdot e^Y$ with $X \in \mathfrak{g}, Y \in V$. Compute the differential $D\Phi_{0,0}$.

Exercise II. (Harder) What is the differntial $D\Phi_{X_0,Y_0}$ at an arbitrary $X_0 + Y_0 \in \mathfrak{g} + V$?

If you have trouble, ask.