Math 4500 Warmup #22, due 3/22/2017

Name: Student Number:

Read Sections 1.3.3 in Hall2015, 8.7 in Stillwell about simply connected spaces and coverings.

Remember the 1/2 hour rule. Hand in a version along with your Homework for next week.

Exercise I. Show that any $g \in SU(2)$ can be written as

$$g = r(\psi)t(\phi)r(\theta) = \begin{bmatrix} e^{i\psi/2} & 0 \\ 0 & e^{-i\psi/2} \end{bmatrix} \cdot \begin{bmatrix} \cos\phi/2 & i\sin\phi/2 \\ i\sin\phi/2 & \cos\phi/2 \end{bmatrix} \begin{bmatrix} e^{i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{bmatrix}$$

How "*unique*" is this presentation? What are the images of these matrices in $SO(3)$ under the adjoint action?

Hint: Look up the material at the beginning of the semester; you may quote the results presented.

Exercise II. Let $\left\{ e_n := \frac{x^{N-n}y^{N+n}}{\sqrt{(N-n)!(N+n)!}} \right\}$ be a basis of $V(N)$. Compute the matrix entry $f_{e_0,e_0}(g)$ in the coordinates given by Exercise I.

Hint: Check the dependence on $\psi,\ \theta$ using the fact that the representation is unitary for the inner product $\langle p, q\rangle = \partial_p(\overline{q})(0)$.

If you have trouble, ask.