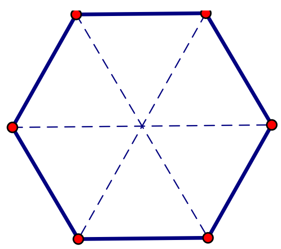
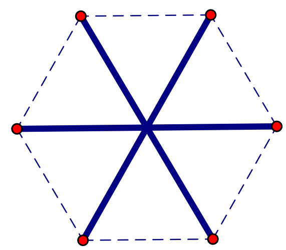
Math 4550 Homework #10

Problems due in class Wednesday, November 28: Read Chapter 4 in my book. Extra credit if you build some interesting examples.

1. The Figure A below, a bar and cable tensegrity in the plane, is not rigid in the plane. The idea is to prove that with absolutely NO numerical calculations. (Five points will be taken off if any such calculations are used.) Just draw pictures and appeal to the results in class.



A

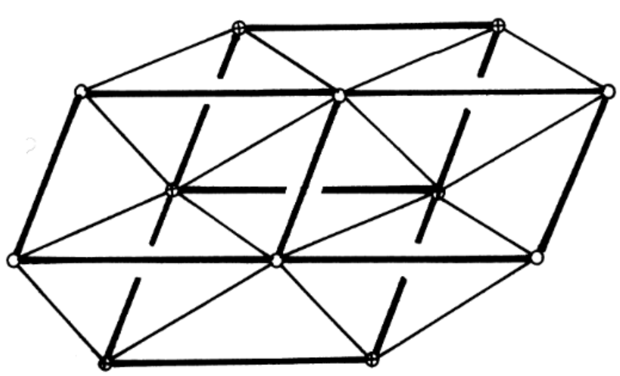


B

   (a) Show that the hexagon in Figure B with cables on its edges and struts for its long diagonals is super stable. You can prove that a convex quadrilateral with cables on its boundary and struts for its diagonals is super stable, and add appropriate such quadrilaterals to show the super stability of Figure B using the symmetry. You can use the three criteria to show super stability.

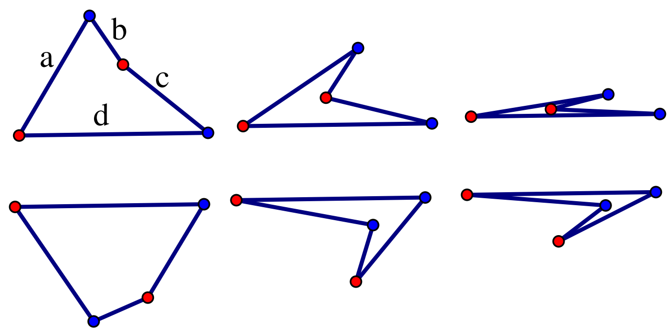
   (b) Use the figure of Problem 4 in Homework #8 to provide an example that is a flex of the tensegrity in Figure A. Use the result in Part (a) to show that the bar and cable conditions are preserved.

2. (a) The following Figure shows a 3-dimensional half-octahedral tetrahedral truss that is commonly used for supporting roofs. Prove that it is generically rigid. Hint: You can use the Henneberg construction in Section 3.2.4 in my book.



   (b) Show that the half-octahedral tetrahedral truss above is not infinitesimally rigid in $\mathbb{R}^3$. Hint: Look for a non-zero equilibrium stress.

3. (a) Suppose you have a quadrilateral in the plane whose bar lengths, in order, are a, b, c, d as shown, and $a - b + c - d = 0$. Show that either the quadrilateral lies on a line, or it is embedded as in the Figure below. Hint: If some pair of edges cross, use the triangle inequality to get a contradiction.



(b) Consider a realization of the graph $K(3,3)$ in the line as in the Figure below, where one partition of the vertices, say the blue vertices, are separated along the line from the other partition, the red vertices. Show that this bar framework is globally rigid in the plane. Hint: Use Part a) and the theorem that says that the graph $K(3,3)$ does not have a topological embedding in the plane.