Math 4550 Homework # 4: Solutions

1. The following is a stress matrix Ω of 5 points of a configuration in an Euclidean space.

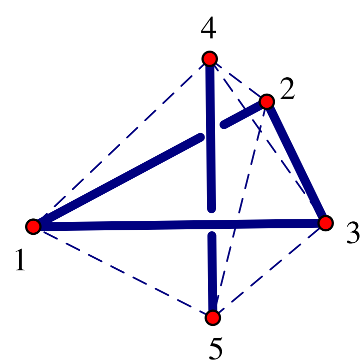
$$\Omega = \begin{bmatrix} 4 & 4 & 4 & -6 & -6 \\ 4 & 4 & 4 & -6 & -6 \\ 4 & 4 & 4 & -6 & -6 \\ -6 & -6 & -6 & 9 & 9 \\ -6 & -6 & -6 & 9 & 9 \end{bmatrix}$$

   (a) Verify that Ω is stress matrix. Ω is symmetric, with row and column sums 1. Thus it is a stress matrix for a tensegrity.

   (b) What is the rank of Ω? What is the dimension of the corresponding universal configuration? Since each row a non-zero multiple of every other row, it is of rank 1. Thus it is a stress matrix for a universal configuration of a tensegrity in $\mathbb{R}^d$ with $d = n-1 = 4-1 = 3$.

   (c) Which off-diagonal entries correspond to cables and which to struts.The $(1,2),(1,3),(2,3),(5,6)$ members are struts (with a negative stress, and positive matrix entry), and the $(1,5),(1,6),(2,5),(2,6),(3,6)$ members are cables.

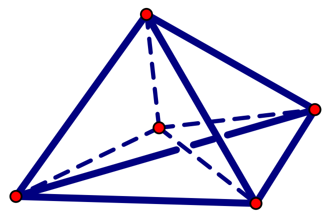
   (d) Note that the three columns and rows $1,2,3$ can be permuted, and the two columns and rows $4,5$ can be permuted without changing the matrix. Draw a picture of the 5 points with cables and struts indicated that is in equilibrium with respect to the stress indicated by Ω, and such that points can be rigidly permuted the same way as the rows and columns.



   (e) Show that $\Omega = A^tA$ where A a 1-by-5 matrix. A^t is A transpose.

$$\Omega = \begin{bmatrix} 2 \\ 2 \\ 2 \\ -3 \\ -3 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 & -3 & -3 \end{bmatrix}$$

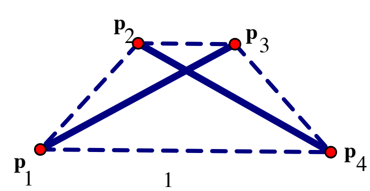
2. The Figure below shows a tensegrity in $\mathbb{R}^3$, where the struts form a regular tetrahedron and the center point is at the center of gravity of the tetrahedron. Find a nice symmetric equilibrium stress for this universally rigid tensegrity, as the figure indicates the cables and struts. Try to use whole integers for the stresses, and take advantage of the symmetry.



$$\Omega = \begin{bmatrix} 16 & -4 & -4 & -4 & -4 \\ -4 & 1 & 1 & 1 & 1 \\ -4 & 1 & 1 & 1 & 1 \\ -4 & 1 & 1 & 1 & 1 \\ -4 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The first row/column corresponds to the central vertex.

3. The vertices of the following figure form an isosceles trapezoid, where $|\mathbf{p}_1 - \mathbf{p}_4| = 3|p_2 - \mathbf{p}_3|$. Find all the equilibrium stresses, and the stress matrix when $\omega_{14} = 1$. What happens to the stress if the trapezoid is not isosceles but still $|\mathbf{p}_1 - \mathbf{p}_4| = 3|p_2 - \mathbf{p}_3|$?



$$\Omega = \begin{bmatrix} 1 & -3 & 3 & -1 \\ -3 & 9 & -9 & 3 \\ 3 & -9 & 9 & -3 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

This follows from $\omega_{12} = \omega_{34}$, $\omega_{13} = \omega_{24}$ by symmetry, $\omega_{12} = -\omega_{13}$ from projection into the y-axis, and $\omega_{12} + 2\omega_{13} + 3\omega_{14} = 0$ from the length and symmetry condition and equilibrium in the x-axis at vertex 1.

When the figure is sheared fixing horizontal lengths, the stress is the same, since that is an affine (linear) transformation.