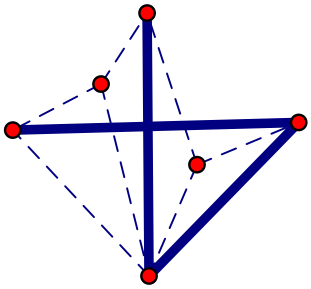
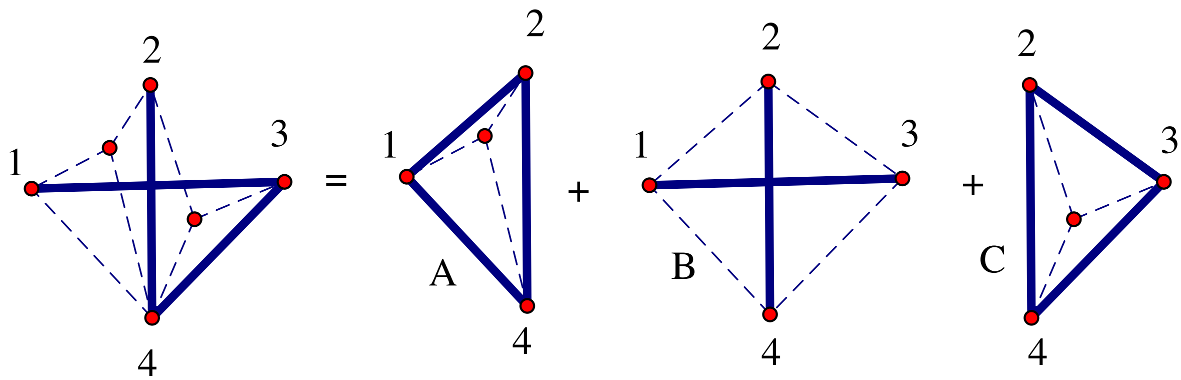
Math 4550 Homework # 5: Solutions

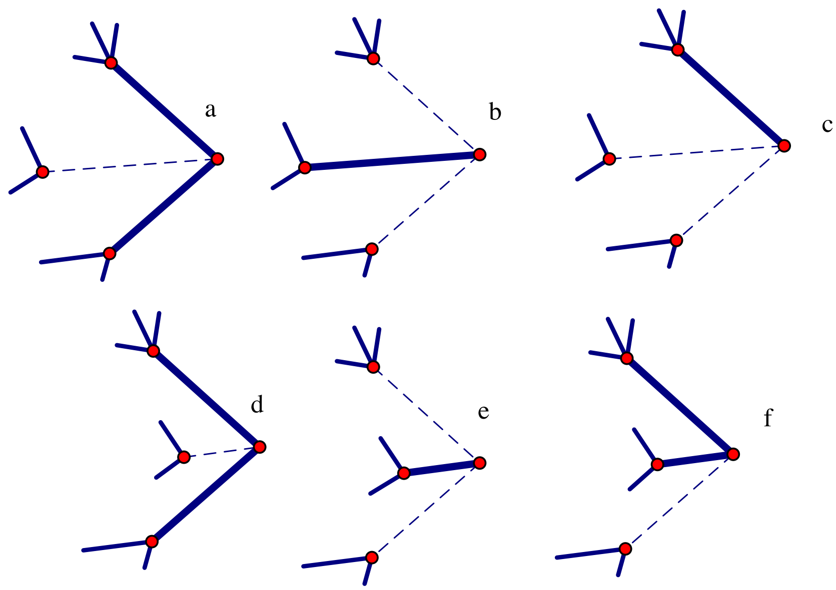
1. Show that the following tensegrity in the plane is universally rigid (i.e. superstable)?



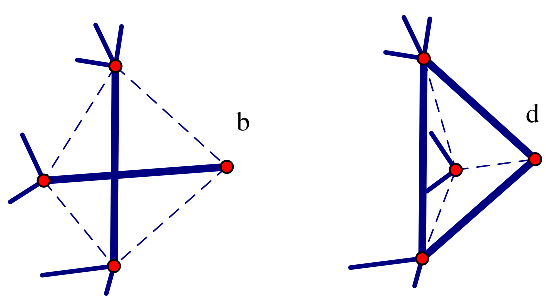
Add the following 3 tensegrites, A, B, C with their max rank PSD stresses so that the stresses on members $(1,2)$ and $(2,3)$ cancel. The remaining stresses on members $(1,4)$ and $(3,4)$ must be as indicated for there to be equilibrium at vertices 1 and 3. The vertices of the tensegrities A and C each overlap with the vertices of tensegrity B at 3 non-colinear points, so the sum tensegrity stress is of maximal rank, namely $6-3=3$.



2. In each of the following portions a planar tensegrity, indicate which ones can be superstable by considering the cable/strut designation at those vertices and indicate why. (Hint: any equilibrium stress at each of these 3-valent vertices is unique up to scaling.)



Figures b and d could be part of a tensegrities as shown and be super stable. In case a, if the vertex's two adjacent struts are fixed in length and rotated up into 3-space, the cable must decrease in length, because of case b it cannot increase in length or stay the same length. So Figure a cannot be part of a super stable tensegrity. Figure e is similar, where the strut could increase in length. Figures c and f are not in equilibrium.



3. In dimension 3, suppose that a superstable tensegrity is such that each edge is a cable or a strut, but not both. Let c be the minimum length of the cables and s the maximum length of the struts. Choose your favorite tensegrity and calculate the ratio $c/s = \rho$. I will give the most points to the person whose tensegrity has the largest value of ρ. Good luck, but make sure your tensegrity is superstable.

   The following, from the solution to Problem 1d in Homework 4, is the best that I could do. The distance of vertex 1, 2, or 3 to the center of the triangle is $1/\sqrt{3}$, where the the strut lengths are 1, then the cable lengths are $\rho = \sqrt{1/3 + 1/4} = \sqrt{7/12} = 0.7637\ldots$.