Math 4550 Homework #7

Problems due in class Friday, November 3: Read Section 3.2 in my book. Extra credit if you build some interesting tensegrities.

Consider the planar a-by-b bar grid of squares in the plane, as below:

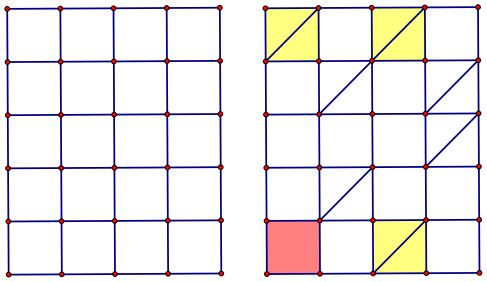

Figure 1: This is a bar framework of $a = 5$ rows of squares, and $b = 4$ columns of squares. The framework on the right has $c = 8$ of the squares braced with diagonal bars.

1. For an a-by-b bar grid of squares in the plane with c braced squares, calculate n the number of vertices and m the number of bars in the final braced framework. What is the minimum value for c, in order for the braced framework to be infinitesimally rigid?

2. Can the framework on the right in Figure 1 be infinitesimally rigid in the plane? Why?

3. Show that if 3 corners of a rectangle in the grid are braced so is the fourth. For example, in the grid on the right, the three yellow squares are braced, so the red square is also braced.

4. If a braced grid framework is rigid, so is the grid you get by permuting the rows or permuting the columns. Permute the rows and permute the columns in the braced grid on the right in Figure 1 so that there is a horizontal line that separates two sets of braced squares, and similarly there is vertical line that separates the same two sets of squares. Use that to find a non-trivial flex of the whole framework. For extra credit, find a corresponding flex of the given framework on the right.

5. If you have a braced grid as above, consider the graph with two sets of vertices, one corresponding the rows of squares and another corresponding to the columns of squares. Place an edge between the row vertex and the column vertex if the corresponding square is braced. What is the relation between the connectivity of that graph and the rigidity of the braced framework?

6. Call G *Laman graph* if $m = 2n - 3$, where n is the number of vertices of G, m is the number of edges of G, and G is generically rigid in $\mathbb{R}^2$. Suppose that all the vertices of a Laman graph G have degree 3, i.e. each vertex is adjacent to exactly 3 edges. What is the minimum number of vertices in such a Laman graph?

7. Show that the following are Laman graphs by doing Hennenberg operations or adding a vertex of degree 2 starting with a triangle.

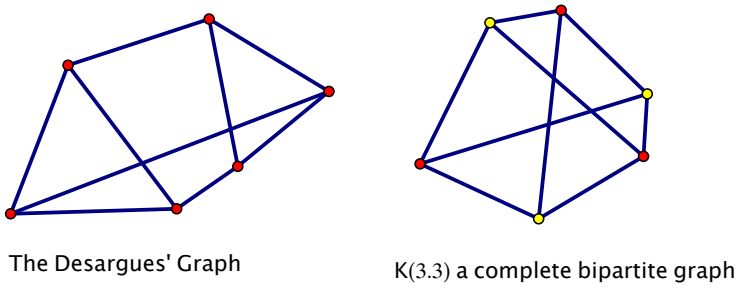

Figure 2

8. In the proof of Laman's Theorem, we started with a vertex of degree 3, removed it, and joined one pair of the adjacent vertices to get a graph with one fewer vertices. Find a Laman graph and a degree 3 vertex, such that when it is removed any pair of the adjacent vertices can be joined with a bar to get a smaller Laman graph.