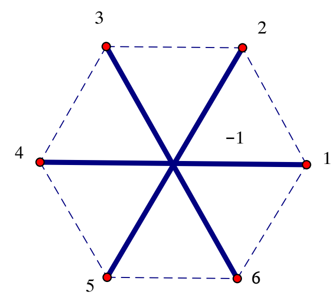
Math 4550 Homework #8-solutions

1. Consider the following tensegrity where the vertices form a regular hexagon with cables on the outside and struts as long diagonal. Compute the equilibrium stress and the stress matrix Ω, where the struts have stress -1.



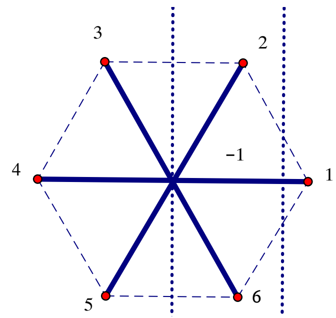
The cables all have stress 2, and the struts all have stress -1. So the stress matrix is:

$$\begin{pmatrix} 3 & -2 & 0 & 1 & 0 & -2 \\ -2 & 3 & -2 & 0 & 1 & 0 \\ 0 & -2 & 3 & -2 & 0 & 1 \\ 1 & 0 & -2 & 3 & -2 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 3 & 0 & 1 & 0 & -2 & 3 \end{pmatrix}.$$

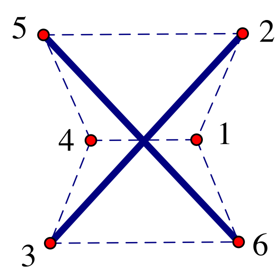
2. Find the eigenvectors and eigenvalues of the the stress matrix Ω in Problem 1 (Hint: Use the symmetry).

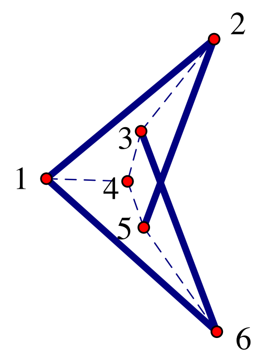
There are 3 0-eigenvalues corresponding to the x and y coordinates of the configuration and the all 1's vector. The trace of Ω is 18, and by symmetry one could conjecture that the other three eigenvalues are the same, and therefore must be equal to 6. For example, the vectors $(1, 0, -1, 1, 0, -1), (0, 1, -1, 0, 1, -1), (1, -1, 0, 1, -1, 0)$ are a basis for the eigenvalue 6.

3. For the two dotted lines in the following Figure, sketch the resulting tensegrity when each of those dotted lines is sent to the line at infinity by a projective transformation.

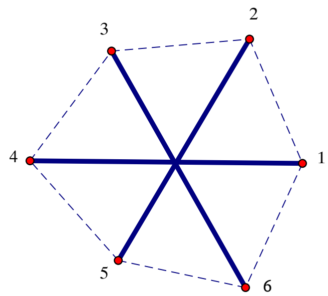


The following are some examples of what the projective images could look like. Note that in the right figure, the 3-vertex cannot be above the $< 1, 2 >$ line, since, if it were, then there would not be equilibrium at the 2 vertex. The struts would be on one side of a line through the 2 vertex and the cables on the other side.





4. In the following figure the hexagon tensegrity has 3-fold rotational symmetry, but not 6-fold symmetry. Is this tensegrity globally rigid in the plane? (Hint: Think push-me, pull-you.)



You can either use the push-me, pull-you method on the symmetric hexagon as in Problem 3, or simply use the symmetry to find another equivalent bar configuration as in the figure below. In other words, rotated the figure and then relabel the vertices.