Problem Set

NAME: ____________________________

§5.2 (Definite Integrals), §5.3 (Indefinite Integrals)

26 June 2018

(1) Find the following indefinite integrals.

(a) $\int (5x^3 - x^{-2} - x^{3/5})\, dx$

(b) $\int \frac{3}{x^{3/2}}\, dx$

(c) $\int \frac{x^2 + 2x - 3}{x^4}\, dx$

(d) $\int 18\cos(3z + 8)\, dz$

(2) If $f''(x) = x^3 - 2x + 1$, $f'(0) = 0$, and $f(0) = 0$, first find f' and then find f.

(3) Evaluate the sums. (You may use a calculator to do simple arithmetic.)

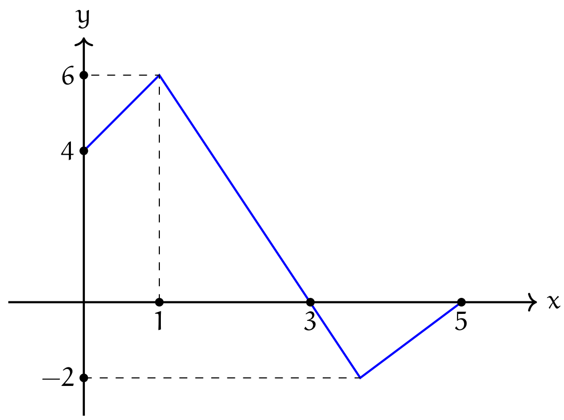
(a) $\displaystyle\sum_{k=1}^{20} 2k+1$

(b) $\displaystyle\sum_{j=1}^{10} j^3+2j^2$

(c) $\displaystyle\sum_{j=101}^{200} j$

(4) Consider the function $f(x) = x^2$ on the interval $[0, 1]$. Find a formula for R_N and compute the area under the graph as a limit. You may use the formula $\displaystyle\sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}$.

(5) Let $f(x)$ be the function plotted below.



Compute the following integrals.

(a) $\int_0^5 f(x)\,dx$

(b) $\int_0^5 |f(x)|\,dx$

(6) Compute the following definite integrals without using the Fundamental Theorem of Calculus. (*Hint: draw a picture.*)

(a) $\int_1^3 |2x-4|\,dx$

(b) $\int_0^\pi \cos x\,dx$

(c) $\int_2^6 \sqrt{4-(x-4)^2}\,dx$

(7) Recall that a function is called **even** if $f(-x) = f(x)$ for all x, and a function is called **odd** if $f(-x) = -f(x)$ for all x. Explain graphically:

(a) If $f(x)$ is an odd function, $\int_{-a}^{a} f(x)\,dx = 0$.

(b) If $f(x)$ is an even function $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

(8) Evaluate $\lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \sqrt{1 - \left(\frac{j}{N}\right)^2}$ by interpreting the limit as an area.